

## Intervention Taxonomy Brief: SpringMath

The goal of this brief is to provide educators with information they can use to evaluate the appropriateness of **SpringMath** for a specific student or group of students who require supplemental and intensive intervention. The brief also may be used to guide decisions about the selection or purchase of a new intervention. We envision that the brief may allow users to examine the extent to which the program aligns to the Taxonomy of Intervention Intensity, a framework used by educators to categorize interventions along key dimensions. The information included in this brief is organized along the seven dimensions of the Taxonomy of Intervention Intensity and can assist educators in answering the following questions:

- Does evidence suggest that this intervention is expected to lead to improved outcomes in the identified area of need (**strength**)?
- Will the group size, duration, structure, and frequency provide sufficient opportunities for students to respond and receive corrective feedback (**dosage**)?
- Does the intervention match the student’s identified needs (**alignment**)?
- Does the intervention assist the student in generalizing target skills to general education or other tasks (**attention to transfer**)?
- Does the intervention include elements of explicit instruction (**comprehensiveness**)?
- Does the student have opportunities to develop the behavior skills necessary to be successful (**behavioral support**)?
- Can the intervention be individualized with a data-based process to meet student needs (**individualization**)?

To learn more about the Taxonomy of Intervention Intensity and find resources to support implementation, visit <https://intensiveintervention.org/taxonomy-intervention-intensity>.

### Program Summary

SpringMath is a web-based MTSS/RTI system for mathematics. Please note: As an RTI system, SpringMath include screening, progress monitoring, and intervention however, NCII has only reviewed the intervention component for the purposes of the Academic Intervention Tools Chart.

## Exhibit 1. Program Information

Features of program implementation	Program recommendations
Grade level(s)	K–12
Group size	No more than six students per group or individual one on one. Classwide intervention is available when a majority of the students are below targets.
Intervention length	15-minute sessions
Frequency	4–5 days per week
Session duration	Groups adjust weekly based on student performance. Intervention continues until children meet exit criteria.
Cost	\$8.95-\$15 per student, which includes universal screening, data interpretation and summary reports, classwide intervention where needed, identification of students for intensified instruction via small-group or individual intervention, progress monitoring, teacher dashboard to summarize growth and organize student-level data for problem-solving meetings and parent-teacher conferences, coach dashboard to direct in-class coaching support where needed, and automated program evaluation.
Training	Initial onboarding training required at a cost of \$895. Ongoing virtual and/or on-site coaching is available from our coaching team. An extensive support portal with video how-to's; support documents; and access to assessments, instructional calendars, lesson plans, word problems, games, presentations, and other materials to facilitate implementation are included at no cost. Bimonthly coach cohort meetings and a monthly newsletter are available to all subscribers at no added cost.

## Evidence of Taxonomy of Intervention Intensity Dimensions

The following section presents definitions for the Taxonomy of Intervention Intensity dimensions and a summary of intervention-specific evidence for each dimension. The evidence comes from the intervention's vendor or developer. It is accurate as reported to the National Center on Intensive Intervention (NCII); it was not independently verified by NCII. Additional program evidence can be found on the [NCII Tools Chart](#) and might appear on the [What Works Clearinghouse](#). For specific questions about the content, contact the publisher at [www.springmath.com](http://www.springmath.com) or Amanda VanDerHeyden at [amandavande@gmail.com](mailto:amandavande@gmail.com).

### Taxonomy Dimension: Strength

*Strength tells us how well the program works for students with intensive intervention needs, expressed in terms of effect sizes. Effect sizes greater than 0.25 indicate an intervention has value in improving outcomes. Effect sizes of 0.35 to 0.40 are moderate, and effect sizes of 0.50 or larger are strong (preferred).*

Exhibit 2 provides the effect sizes for students in need of intensive intervention organized by domain and subdomain. These effect size data are calculated on low achieving participants, those falling at or below the 20th percentile on pretest measures of achievement. If available, additional effect sizes for disaggregated data can be found on the NCII Tools Chart.

**Exhibit 2. SpringMath Effect Sizes for Students ≤20th Percentile by Domain and Subdomain**

Domain	Subdomain	Outcome measures	Effect size <sup>a</sup>
Mathematics	<ul style="list-style-type: none"> <li>▪ Math Computation               <ul style="list-style-type: none"> <li>• Fact Families for Addition and Subtraction 0–20</li> <li>• 3-Digit Addition and Subtraction With and Without Regrouping</li> <li>• Fact Families for Multiplication and Division 0–12</li> </ul> </li> </ul>	SpringMath <sup>b</sup> measures from Grades 2–4	0.49
Mathematics	<ul style="list-style-type: none"> <li>▪ Math Computation</li> </ul>	Monitoring Basic Skills Progress—Computation	-0.22
Mathematics	<ul style="list-style-type: none"> <li>▪ Math Concepts</li> <li>▪ Math Application</li> </ul>	Monitoring Basic Skills Progress—Concepts and Applications	0.34
Mathematics	<ul style="list-style-type: none"> <li>▪ Math Computation</li> </ul>	3-digit addition & subtraction with & without regrouping (Grade 4)	0.56
Mathematics	<ul style="list-style-type: none"> <li>▪ Math Computation</li> </ul>	Multiplication 0-12 (Grade 4)	1.11*
Mathematics	<ul style="list-style-type: none"> <li>▪ Math Computation</li> </ul>	Fact Families Multiplication & Division 0-12 (Grade 4)	0.9*
Mathematics	<ul style="list-style-type: none"> <li>▪ Math Computation</li> </ul>	Fact Families Multiplication & Division 0-12 (Grade 5)	0.63
Mathematics	<ul style="list-style-type: none"> <li>▪ Math Computation</li> </ul>	Multiply 2 by 3 digits without regrouping (Grade 5)	0.66
Mathematics	<ul style="list-style-type: none"> <li>▪ Fractions</li> </ul>	Simplify Fractions (Grade 5)	0.98*
Mathematics	<ul style="list-style-type: none"> <li>▪ Comprehensive</li> </ul>	Mississippi Curriculum Test, 2nd Ed. (Grade 4)	0.79*
Mathematics	<ul style="list-style-type: none"> <li>▪ Comprehensive</li> </ul>	Mississippi Curriculum Test, 2nd Ed. (Grade 5)	-0.05

<sup>a</sup> To ensure comparability of effect size across studies, NCII uses a standard formula to calculate effect sizes across all studies and outcome measures—Hedges *g*, corrected for small-sample bias. <sup>b</sup> *SpringMath* measures can be referenced as follows: Education Research & Consulting, Inc. (2013). *SpringMath* (version 2.2.1) [Computer Software]. Sourcewell Technology. <https://www.sourcewelltech.org/student-achievement/math-intervention-spring-math>.

\*  $p \leq .05$ .

### **Taxonomy Dimension: Dosage**

*Dosage is the number of opportunities a student has to respond or practice and receive corrective feedback. Dosage may be impacted by the size of the instructional group, the number of minutes each session lasts, the number of student-teacher interactions built into lessons, and the number of sessions provided per week.*

Assuming a group size of no more than six students, each student in the group has an estimated 76 opportunities to respond and receive corrective feedback.<sup>1</sup>

### **Taxonomy Dimension: Alignment**

*Alignment (Exhibit 3) focuses on how well the program (a) addresses the target student’s full set of academic skill deficits, (b) does not address skills the target student has already mastered (extraneous skills for that student), and (c) incorporates a meaningful focus on grade appropriate curricular standards.*

### **Exhibit 3. Alignment With Content Areas Addressed**

<b>Instructional grade level(s)</b>	<b>Content area addressed</b>	<b>Skill strands</b>
Kindergarten	Counting and Cardinality	<ul style="list-style-type: none"><li>▪ Object-number correspondence in counting, rote counting, number naming to 10 and 20.</li><li>▪ Ordinal understanding of numbers to 10 and 20.</li><li>▪ Quantity discrimination with dot sets of up to 10 and 20.</li></ul>
Kindergarten	Number and Operations in Base 10	<ul style="list-style-type: none"><li>▪ Make quantities to 20 using manipulatives and drawings, make a quantity one less, make a quantity one more, make an equivalent quantity.</li><li>▪ Use addition/subtraction expressions to make quantities to 20.</li></ul>
Kindergarten	Operations and Algebraic Thinking	<ul style="list-style-type: none"><li>▪ Change quantities to make 10.</li><li>▪ Find the “double” quantities.</li><li>▪ Make quantities to 5 and 10 using manipulatives and drawings.</li><li>▪ Use addition/subtraction expressions to make numbers 5 to 10.</li><li>▪ Add and subtract using numbers 0 to 5 with manipulatives.</li></ul>

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<sup>1</sup> This number represents the average opportunities to respond per day of intervention. All students in a group respond using their own set of printed materials to every response opportunity. Children are recommended for intensified instruction (Tier 2 or Tier 3) either directly from universal screening or (more commonly) following classwide intervention. Once identified for intervention, students are given a diagnostic assessment to identify the optimal skill target and intervention tactic. If more than one student who is available at the same time needs the same intervention, then those students are recommended for a small group. Groups are adjusted weekly so that each child’s optimal intervention fit is maintained. Thus, there is no functional difference between Tier 2 and Tier 3 interventions in SpringMath except that Tier 3 interventions may be delivered in any given week as part of a small group with no more than six children, in which case we call it a Tier 2 intervention. It is possible (and commonly occurs) that a student may be part of a small-group one week and then receive individual intervention the following week during intervention. SpringMath views the Tier 2 option as a mechanism to gain efficiency in intervention delivery, but that is desirable only if the intervention-child need alignment is maintained, so all Tier 2 and Tier 3 interventions are matched with students based on diagnostic assessment and weekly progress monitoring during intervention. The same protocols are used in Tier 2 and Tier 3.

Instructional grade level(s)	Content area addressed	Skill strands
Grade 1	Operations and Algebraic Thinking	<ul style="list-style-type: none"> <li>▪ Add to 20.</li> <li>▪ Subtract 0–20.</li> <li>▪ Create equivalent expressions with addition and subtraction.</li> <li>▪ Use manipulatives, drawings, and number lines to solve addition and subtraction.</li> <li>▪ Solve fact families for addition and subtraction 0–20.</li> <li>▪ Solve word problems that require multistep solutions and require child to reach conclusions about quantity using addition and subtraction, solving for unknowns, and creating equivalent expressions.</li> <li>▪ Use associative property of addition to solve for sums using manipulatives, drawings, and numerical expressions.</li> </ul>
Grade 1	Number and Operations in Base 10	<ul style="list-style-type: none"> <li>▪ Quantity comparison with numbers to 20–99 and 100–999.</li> <li>▪ Add and take tens to quantities up to 85.</li> <li>▪ Decompose 10s and 1s to add quantities.</li> <li>▪ Create equivalent expressions using 10s and 1s and counting with manipulatives to check.</li> <li>▪ Compose 10s to create equivalent expressions.</li> </ul>
Grade 2	Operations and Algebraic Thinking	<ul style="list-style-type: none"> <li>▪ Add to 20.</li> <li>▪ Subtract 0–20.</li> <li>▪ Solve fact families for addition and subtraction 0–20.</li> <li>▪ Create equivalent addition and subtraction problems using place value properties.</li> <li>▪ Create equivalent problems using associative and near-easy problems.</li> <li>▪ Quantity comparison of sums and differences to 20.</li> <li>▪ Make equivalent sets with manipulatives and express as addition.</li> <li>▪ Model numerical quantities up to 50 on a number line as repeated addition of 1s and 10s, 10 expressed as adding 10 sets of 1, and a hundred expressed as adding 10 sets of 10.</li> </ul>
Grade 2	Number and Operations in Base 10	<ul style="list-style-type: none"> <li>▪ Quantity comparison with numbers 101–999 and 1,000–9,999.</li> <li>▪ Solve two-digit addition and subtraction problems via expanded notation, decomposing and composing 10s and 100s.</li> <li>▪ Take and add 10s and 100s.</li> <li>▪ Create equivalent expressions using multiple addends.</li> </ul>
Grade 3	Operations and Algebraic Thinking	<ul style="list-style-type: none"> <li>▪ Solve fact families for addition and subtraction 0–20.</li> <li>▪ Solve word problems requiring solve for unknown setup; converting more challenging problems to easier problems to solve operations via expanded notation; and articulate patterns, estimate, think aloud, and justify answers using what understanding of related operations.</li> <li>▪ Multiply 0–12.</li> <li>▪ Solve multiplication problems as repeated addition, using arrays, verbally describing quantities as sets, and solving word problems.</li> <li>▪ Divide 0–12.</li> </ul>

Instructional grade level(s)	Content area addressed	Skill strands
		<ul style="list-style-type: none"> <li>▪ Articulate division as finding an unknown factor, partitioning sets using graphics (visual representation), and solving word problems.</li> <li>▪ Solve fact families for multiplication and division 0–12.</li> <li>▪ Solve word problems setting up and solving for an unknown while articulating the relationship between multiplication and division.</li> <li>▪ Demonstrate commutative, associative, and distributive property in solving for unknowns and creating equivalent expressions.</li> </ul>
Grade 3	Number and Operations in Base 10	<ul style="list-style-type: none"> <li>▪ Add and subtract three-digit numbers using place value understanding.</li> <li>▪ Use expanded notation to add and subtract with three-digit numbers and to compose and decompose 10s and 100s.</li> <li>▪ Use expanded notation to solve for unknowns with multidigit addition and subtraction problems.</li> <li>▪ Multiply one-digit by two- or three-digit numbers using place value understanding.</li> <li>▪ Divide one-digit into two- or three-digit dividend using place value understanding.</li> </ul>
Grade 3	Number and Operations: Fractions	<ul style="list-style-type: none"> <li>▪ Compare fraction quantities for fractions with like denominators.</li> <li>▪ Understand fraction base units.</li> <li>▪ Represent fraction quantity as repeated addition of the base unit fraction on a number line and in repeated addition expressions.</li> <li>▪ Locate fraction quantities on number lines.</li> <li>▪ Understand fraction as a special case of division.</li> <li>▪ Place fraction quantities on a number line using the denominators 2, 4, and 8.</li> </ul>
Grade 4	Operations and Algebraic Thinking	<ul style="list-style-type: none"> <li>▪ Solve fact families for multiplication and division 0–12.</li> <li>▪ Create equivalent multiplication expressions using common factors.</li> <li>▪ Quantity comparison fractions, whole numbers, and decimals.</li> </ul>
Grade 4	Number and Operations in Base 10	<ul style="list-style-type: none"> <li>▪ Quantity comparison with decimals to the hundredths.</li> <li>▪ Use expanded notation and mental math strategies to estimate solutions (and justify estimations) based on place value understanding with all four operations.</li> <li>▪ Add and subtract with decimals to the hundredths.</li> <li>▪ Multiply one-digit by two- or three-digit numbers using place value (map on a number line, use expanded notation, create equivalent expressions).</li> <li>▪ Multiply two-digit by two-digit numbers using place value understanding (map on a number line, use expanded notation, create equivalent expressions).</li> <li>▪ Divide one-digit into two- or three-digit dividend using place value understanding.</li> <li>▪ Convert expressions to solving for an unknown factor plus an addend if needed and locate quantity on a number line.</li> </ul>

Instructional grade level(s)	Content area addressed	Skill strands
Grade 4	Number and Operations: Fractions	<ul style="list-style-type: none"> <li>▪ Place fractions on a number line with quantities <math>&gt; 1</math> and challenging denominators of 1, 2, 3, 4, 5, 6, 8, and 10.</li> <li>▪ Quantity comparison with fractions with unlike denominators.</li> <li>▪ Convert fractions to decimals and decimals to fractions.</li> <li>▪ Determine when fractions or decimals are the best unit with which to solve real-world problems.</li> <li>▪ Add and subtract mixed numbers with like denominators and regrouping.</li> <li>▪ Place mixed number and fraction quantities on a number line.</li> <li>▪ Create equivalent quantities with fractions using addition of fraction units and multiplication of fraction units.</li> <li>▪ Demonstrate fraction equivalence in simplified form.</li> <li>▪ Estimate and make quantity comparisons with fractions and decimals and solve related word problems.</li> </ul>
Grade 5	Operations and Algebraic Thinking	<ul style="list-style-type: none"> <li>▪ Use understanding of associative property and distributive property to create equivalent expressions with addition (associative) and multiplication (associative and distributive).</li> <li>▪ Determine quantity using understanding of operations without conducting actual operations.</li> <li>▪ Use understanding of numerical patterns and operations to create equivalent expressions with various number representations.</li> <li>▪ Quantity comparison whole numbers, fractions, decimals, and percentages.</li> </ul>
Grade 5	Number and Operations in Base 10	<ul style="list-style-type: none"> <li>▪ Quantity comparison for whole numbers, fractions, and decimals.</li> <li>▪ Create equivalent expressions using repeated addition, multiplication, and division to reflect place value understanding.</li> <li>▪ Use mental math strategies to compare quantities and to estimate problem solutions with decimals.</li> <li>▪ Solve fact families for multiplication and division 0–12.</li> <li>▪ Multiply two-digit by two-digit numbers using place value understanding.</li> <li>▪ Divide two2-digit into three- or four-digit dividend using place value understanding.</li> <li>▪ Add and subtract with decimals to the hundredths.</li> <li>▪ Multiply and divide with decimals to the hundredths.</li> <li>▪ Proof division algorithms with decimals using fraction quantities to illustrate problem solution (after verifying understanding of conversion of decimals to fractions and vice versa) and understanding of addition, subtraction, multiplication, and division to estimate then verify decimal quantities following related operations.</li> </ul>
Grade 5	Number and Operations: Fractions	<ul style="list-style-type: none"> <li>▪ Create equivalent fractions that share least common denominator.</li> <li>▪ Simplify fractions.</li> <li>▪ Convert improper fractions to mixed numbers.</li> <li>▪ Convert mixed numbers to improper fractions.</li> <li>▪ Add and subtract fractions with unlike denominators.</li> </ul>



Instructional grade level(s)	Content area addressed	Skill strands
		<ul style="list-style-type: none"> <li>▪ Simplify fractions.</li> <li>▪ Multiply and divide proper and improper fractions.</li> <li>▪ Quantity comparison with fractions, whole numbers, and decimals.</li> <li>▪ Quantity comparison with fractions, whole numbers, decimals, and percentages.</li> <li>▪ Proof proportion quantity on a number line and convert between proportions to make problems easier to solve.</li> <li>▪ Solve word problems using understanding multiplication of fraction quantities and their meaning as scaling by a factor.</li> </ul>
Grade 6	The Number System	<ul style="list-style-type: none"> <li>▪ Mixed whole number operations.</li> <li>▪ Create equivalent proportions, find least common denominator, and simplify fractions.</li> <li>▪ Add, subtract, multiply, and divide with fractions and mixed numbers.</li> <li>▪ Add, subtract, multiply, and divide with decimals to tenths and hundredths.</li> <li>▪ Find percent of a whole number.</li> <li>▪ Graph points in a coordinate plane.</li> <li>▪ Quantity comparison with negative numbers (locate integers on a number line, understand meaning of zero in real-world problems).</li> </ul>
Grade 6	Expressions and Equations	<ul style="list-style-type: none"> <li>▪ Use order of operations (excluding exponents) conventions to solve equations.</li> <li>▪ Substitute whole number to solve equations.</li> <li>▪ Collect like terms (requires adding to consolidate variables and numbers and subtracting to consolidate like variables in an expression).</li> <li>▪ Distributive property of expression.</li> <li>▪ Solve word problems that require using inverse operations of addition/subtraction and multiplication/division.</li> <li>▪ Use inverse operations to create equivalent expressions.</li> <li>▪ Determine which expression is greater than or less than another expression and change one expression to make expressions equivalent.</li> </ul>
Grade 6	Ratios and Proportional Relationships	<ul style="list-style-type: none"> <li>▪ Find percent of a whole number.</li> <li>▪ Linear change between independent and dependent variables in the context of graphing points in coordinate plane.</li> </ul>
Grade 7	Ratios and Proportional Relationships	<ul style="list-style-type: none"> <li>▪ Add, subtract, multiply, and divide with fractions and mixed numbers.</li> <li>▪ Solve for missing value in two equivalent algebraic proportions.</li> <li>▪ Find percent of a whole number.</li> <li>▪ Solve equations with percentages.</li> <li>▪ Solve missing value in a percentage problem.</li> <li>▪ Complex fractions.</li> <li>▪ Convert decimals to fractions and fractions to decimals.</li> </ul>



Instructional grade level(s)	Content area addressed	Skill strands
Grade 7	The Number System	<ul style="list-style-type: none"> <li>▪ Add, subtract, multiply, and divide integers of varied sign.</li> <li>▪ Inverse operations for addition, subtraction, multiplication, and division.</li> <li>▪ Quantity comparison with negative numbers.</li> <li>▪ Proof positive and negative quantities on a number line.</li> <li>▪ Understand and anticipate results when multiplying and dividing with positive and negative quantities.</li> </ul>
Grade 7	Expressions and Equations	<ul style="list-style-type: none"> <li>▪ Use order of operations conventions to solve equations.</li> <li>▪ Translate verbal expressions into math equations.</li> <li>▪ Solve two-step equations.</li> <li>▪ Solve step-step equations with fractions.</li> </ul>
Grade 8	The Number System	<ul style="list-style-type: none"> <li>▪ Add, subtract, multiply, and divide with fractions and mixed numbers.</li> <li>▪ Add, subtract, multiply, and divide integers of varied sign.</li> </ul>
Grade 8	Expressions and Equations	<ul style="list-style-type: none"> <li>▪ Collect, distribute, and factor to simplify exponent expressions with positive and negative exponents attached to variables, whole numbers, and fractions expressed as equations using addition, subtraction, multiplication, and division.</li> <li>▪ Proof exponent quantity, compare exponent quantities, and use operations to create equivalent quantities with exponents.</li> <li>▪ Use scientific notation with exponents to create equivalent quantities, to make quantity comparisons, and to make quantity statements true.</li> <li>▪ Solve for slope and intercept using linear function.</li> <li>▪ Use distributive property to simplify expressions.</li> <li>▪ Collect like terms to simplify expressions.</li> <li>▪ Simplify expressions.</li> <li>▪ Use order of operations conventions to solve equations (including exponents).</li> </ul>
Grade 8	Functions	<ul style="list-style-type: none"> <li>▪ Solve for slope and intercept using linear function.</li> <li>▪ Determine if a given point (coordinate pair) can fall on a given line.</li> <li>▪ Use linear combinations to solve equations.</li> <li>▪ Substitute equation to solve linear equations.</li> <li>▪ Comparison method to solve linear equations.</li> </ul>
Grades 9–12 <sup>a</sup>	Number and Quantity	<ul style="list-style-type: none"> <li>▪ Mixed operations.</li> <li>▪ Create equivalent fractions that share least common denominator.</li> <li>▪ Simplify fractions.</li> <li>▪ Add and subtract fractions with unlike denominators.</li> <li>▪ Multiply and divide proper and improper fractions.</li> <li>▪ Convert fractions to decimals and decimals to fractions.</li> <li>▪ Add and subtract with decimals to the hundredths.</li> <li>▪ Multiply and divide with decimals.</li> <li>▪ Multiply two-digit by two-digit with decimals.</li> <li>▪ Find percent of a whole number.</li> <li>▪ Algebraic proportions.</li> </ul>

Instructional grade level(s)	Content area addressed	Skill strands
		<ul style="list-style-type: none"> <li>▪ Complex fractions.</li> <li>▪ Add and subtract with integers.</li> <li>▪ Add, subtract, multiply, and divide integers of varied sign.</li> <li>▪ Mixed inverse operations: add, subtract, multiply, and divide.</li> <li>▪ Collect like terms to simplify expressions.</li> <li>▪ Distributive property to simplify expressions.</li> <li>▪ Add and subtract with exponents.</li> <li>▪ Multiply with exponents.</li> <li>▪ Divide with exponents.</li> <li>▪ Simplify expressions.</li> </ul>
Grades 9–12 <sup>a</sup>	Algebra	<ul style="list-style-type: none"> <li>▪ Order of operations (excluding exponents).</li> <li>▪ Solve equations with percentages.</li> <li>▪ Calculate missing value in percentage problem.</li> <li>▪ Translate verbal expressions into math equations.</li> <li>▪ Solve two-step equations.</li> <li>▪ Solve two-step equations with fractions.</li> <li>▪ Order of operations II (includes exponents).</li> </ul>
Grades 9–12 <sup>a,b</sup>	Functions	<ul style="list-style-type: none"> <li>▪ Solve for slope and intercept using linear function <math>y = mx + b</math>.</li> <li>▪ Point on a line.</li> <li>▪ Linear combinations to solve equations.</li> <li>▪ Substitute equation to solve linear equation.</li> <li>▪ Use comparison method to solve systems of linear equations.</li> </ul>

<sup>a</sup> There is planned overlap in skill coverage between grades. All associated prerequisite skills attached to any skill are accessed through diagnostic assessment in planning intervention. We listed the grade-level expected proficiency. In a few cases, our skills very slightly outpace Common Core State Standards by design. In these cases, we build proficiency first on the easier skill ranges and introduce the more challenging iteration only following mastery of the easier subskill. In each case, we introduce the challenging iteration to smooth the cross-grade-level expectations or to provide greater facility in grade-level understandings that build on the skill. For example, in Grade 3, we emphasize factors to 12 (as opposed to 10) to aid in fraction work that is coming in Grade 4. We first build mastery in multiplication 0–9 and then mastery in facts 10–12. In other cases, a below-grade-level skill may be listed; when that happens, it provides an entry point to drill down to prerequisite skills that may be interfering with grade-level success (e.g., mixed fraction operations at Grade 8). In this way, from any grade level, from screening or from classwide intervention, the child’s performance and subsequent assessment can be used to place the child precisely into the right skill for intervention, even if that skill is below grade level. <sup>b</sup> *SpringMath* in Grades 9–12 is intentionally a remedial supplement and therefore targets below-grade-level skills in mathematics.

A [full alignment study](#) was conducted to specify specific Common Core State Standards addressed in *SpringMath* for Grades K–8. Specifically, across grade levels, we provide extensive coverage via assessment and intervention for Counting and Cardinality, Operations and Algebraic Thinking, Number and Operations in Base 10, Number and Operations with Fractions, The Number System, Expressions and Equations, and Ratios and Proportional Relationships. We do not provide assessment and instruction for Measurement and Data, Geometry, and Statistics and Probability.

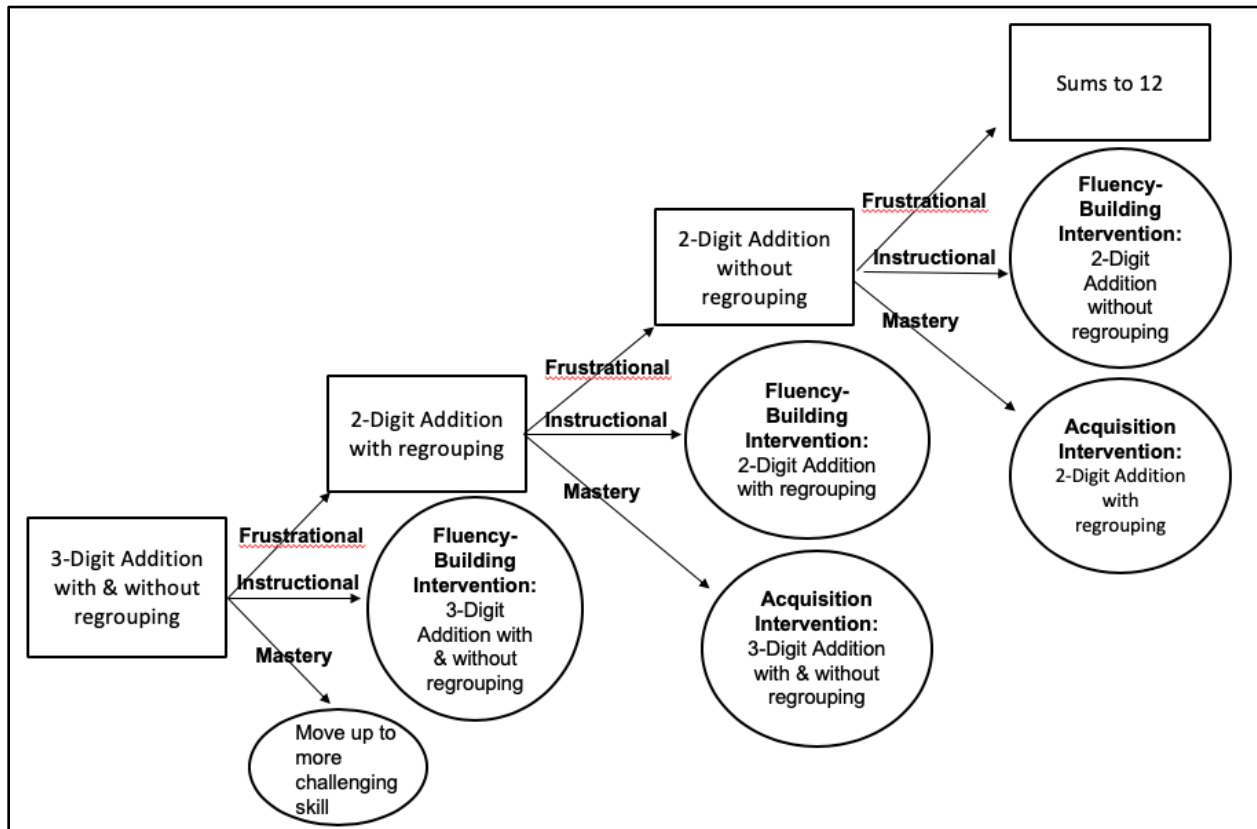
*SpringMath* begins with [universal screening](#) at fall, winter, and spring. Three to four measures are given at each screening occasion and reflect current grade-level skills that differ by season (Grades 9–12 screening measures administered per year). If there is a high base rate of risk at

screening (defined as median score in the frustrational range), then the class is routed into classwide intervention. Weekly progress monitoring data are then harvested using the *SpringMath* rules engine to support correct classwide intervention use and identify children who need intensified instruction in that context (classification accuracy for both static screening and classwide intervention have been reported, VanDerHeyden et al., 2019).

The classwide intervention skill sequence varies by grade level, beginning with below-grade-level foundation skills and progressing to essential grade-level skills. The whole class advances to the next skill based on mastering the first skill and so on. The classwide intervention is a standard protocol fluency-building intervention, which has been evaluated for efficacy (Coddling et al., 2016; VanDerHeyden et al., 2012).

Decision trees (Exhibit 4) are specified that begin with the grade and season screening skills. Each screening skill is the first skill in a given tree and samples back through prerequisite skills, specifying frustrational, instructional, and mastery level performance for each skill. Thus, all 143 skills are contained in hierarchical and independent decision trees for which the screening is the “dropping in” point, closely connected to grade-level content. The diagnostic assessment follows the decision tree to identify the correct intervention skill target and intervention tactic (acquisition or fluency-building).

#### Exhibit 4. Decision Tree Example



Weekly assessment of the skill target and the generalization target (the screening skill) is conducted to monitor progress and advance the intervention content and tactic until the student reaches mastery on the grade-level skill, at which time the student is exited from intervention.

### **Taxonomy Dimension: Teaching to Promote Transfer**

*Attention to transfer is the extent to which an intervention is designed to help students (a) transfer the skills they learn to other formats and contexts and (b) realize connections between mastered and related skills.*

For whole number multiplication (instructional target), three activities designed to explicitly teach for transfer are multiple representations, practice creating equivalent quantities (which we call a “tool skill”) within the context of the skill being taught, and embedding high dosage of opportunities to respond to build fluency and facilitate generalization.

**Activity 1.** Shows multiple ways to solve using mathematical understanding the student has already mastered and provides multiple opportunities for the child to practice solving with immediate corrective feedback until the student is accurate.

During acquisition, providing multiple representations helps students connect understanding of new skills to already mastered understanding/skills, which builds confidence, reduces task difficulty, and facilitates acquisition. Providing multiple representations reduces task difficulty/complexity, improves student engagement, and increases likelihood of correct responding when new skills are introduced, but also cultivates robust learning (i.e., learning that is more likely to be generalized to new stimulus conditions) because the task/stimulus similarities for which the learned solution can be successful are made explicit such that the student can readily recognize the stimulus conditions under which learned skill can be useful (Exhibit 5).

## Exhibit 5. Activity 1 Example

Let's find the solution to  $3 \times 9$  on a number line. We can think of  $3 \times 9$  as 3 sets of 9. Let's count out each set and then find the product or answer of  $3 \times 9$ .

$9 + 9 + 9 = 27$

$9 + 9 + 9 = 27$

How many sets of 9 are in 27?

$3 \times \underline{\quad} = 27$

We can also solve  $3 \times 9$  using an area model.

		9											
		1	2	3	4	5	6	7	8	9	10	11	12
<b>3 Sets of</b>	1	/	/	/	/	/	/	/	/	/			
	2	/	/	/	/	/	/	/	/	/			
	3	/	/	/	/	/	/	/	/	/			
	4												
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	11												
	12												

**Activity 2.** Provides a model and guided practice to create equivalent expressions from multiplication problems. Creating equivalent expressions helps students make challenging problems easier to solve. In this example (Exhibit 6), assisting the student to represent a multiplication problem as repeated addition provides the student another way to solve a multiplication problem for which the student may not readily know the answer. Similarly finding an “easy known” and counting up or down to the correct answer is another solution path that the student can use when encountering unknown problems. Finally, solving for an unknown via the “fill-in the missing number” activity provides practice in how the student can solve for an unknown (which is the general goal of math—to find solutions to real-world problems). In effect, these activities are guided practice to connect skills and content in mathematics (i.e., uncover the coherent nature of mathematics) and transfer the learned skill.

## Exhibit 6. Ways to Approach Multiplication

	Write as Addition	Find the Double or Near Easy	Fill-in Missing Number
$6 \times 7 =$	$6 + 6 + 6 + 6 + 6 + 6 +$ $6 = 42$ Or $7 + 7 + 7 + 7 + 7 + 7 =$ $42$	$6 \times 6 + 6 = 42$ Or $7 \times 7 - 7 = 42$	$6 \times 4 + \underline{\quad} = 42$
$8 \times 9 =$			$8 \times 10 - \underline{\quad} = 72$
$7 \times 8 =$			$7 \times 4 \times \underline{\quad} = 56$
$9 \times 6 =$			$3 \times 3 \times 3 \times \underline{\quad} = 54$
$7 \times 4 =$			$7 \times 2 \times \underline{\quad} = 28$

**Activity 3.** Builds fluency using games to embed high dosages of opportunities to respond. High dosages of opportunities to respond at the instructional level of skill difficulty are one of the most effective ways to promote skill fluency (Exhibit 7). Fluent skills are resistant to forgetting and more likely to be used or modified to solve novel related problems (i.e., more likely to generalize). Thus, building fluency is a powerful way to build robust skills that can endure across time and in learning new, related content.

## Exhibit 7. Fluency Practice

Play a game with the student. You have several options here. Using the two bingo cards below, you can play several games. You can play against the student, using the problems from the day's worksheet to find and cover products on each card. You can simply use a pencil to cross products out and the winner is the first person to cross out all the products on the grid. You can play the game such that the first person to get 4 products in a row either vertically or horizontally is the winner. You can have two students play against each other. Finally, a really fun variation is to make a secret mark in a few places on the grid using a crayola "changeable" marker, the student colors in each square when the square contains the correct product for the multiplication problem you call out. When the secret code is revealed, the student wins the game.

30	35	40	54	63
36	54	64	42	72
56	49	56	81	63
81	48	42	54	36
25	49	45	64	72

### Taxonomy Dimension: Comprehensiveness

*Comprehensiveness is the number of explicit instruction principles the intervention incorporates (e.g., providing explanations in simple, direct language; modeling efficient solution strategies instead of expecting students to discover strategies on their own; providing practice so that students use the strategies to generate many correct responses; and incorporating systematic cumulative review). Additional information can be found within the NCII [Explicit Instruction course content materials](#).*

Acquisition protocols and fluency-building protocols were randomly selected for 46 of 143 skills (32%) and were audited for certain features, some of which are summarized in Exhibit 8. (We would be happy to provide our rating form.) Protocols came from Grades 1–6. All interventions provided scripted instruction for conceptual understanding and procedural instruction, interleaved daily; multiple representations of mathematical concepts and/or equations; explicit and systematic instruction, including modeling, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review. In addition, interventions sampled contained the following features (Exhibit 8) in scripted form with all practice problems provided in the weekly intervention packet. Practice problems are generated using our assessment generator to provide technically equivalent (similarly difficult) problems that differ each time the intervention protocol is generated by *SpringMath*.



## Exhibit 8. Intervention Features

Intervention features	Percentage of interventions using this feature
Explicit proofing of algorithm	100%
More than one way to solve	100%
Connection to previous understanding	100%
Connection to future understanding	100%
A model of correct responding	100%
Immediate corrective feedback	100%
Practice discriminating quantity	98%
Converting quantities	100%
Constructing and/or solving for an unknown	96%
Create an equivalent quantity	100%
Fill in the missing number	95%
Detect, repair, explain errors	13% in conceptual understanding activities; 100% in procedural skill-building
Conceptual practice materials provided?	100%
A game	57% (mostly for fluency-building protocols)
Manipulatives	28%
Graphic representation	85%
Asks child to articulate a rule or pattern, estimate quantity based on understanding of operations and starting quantities, justify solution	100%
Word problem	51% (used in fluency-building protocols)
Solve a more challenging problem type	78%
Goals for improvement w reward system	100%

### ***Dimension: Provide Directions in Clear, Direct Language***

**Activity 1.** Intervention scripts are provided in printable weekly packets. Teacher language is provided in boldface print, step-by-step, to explain how to solve a particular problem. An error correction script also is provided.

### ***Dimension: Model Efficient Solution Strategies***

**Activity 1.** We model (and script for the teacher) correct problem solving prior to assessment and the procedural skill-building portion of daily intervention. Procedural interventions include cover/copy/compare and guided practice intervention types with well-controlled novel practice problems generated within each protocol pack for the week.

**Activity 2.** We provide scripted and modeled activities to build conceptual understanding in the conceptual understanding part of the daily intervention. For example, the teacher follows this script as part of the Guided Practice Intervention for converting improper fractions to mixed numbers (Exhibit 9).

## Exhibit 9. Improper Fractions Example

This value should match the mixed number value we started with. To understand why this works, let's use what we know about multiplication and division.

$$\frac{55}{7} = \frac{7}{7} \times \frac{\quad}{\quad} = \frac{49}{7}$$

This one is not enough.  
How much more is needed?  $\frac{7}{7} \times \frac{7}{7} + \frac{\quad}{\quad} = \frac{55}{7}$

$$\frac{55}{7} = \frac{7}{7} \times \frac{\quad}{\quad} = \frac{56}{7}$$

This one is too much.  
How much too much is this one?  $\frac{7}{7} \times \frac{8}{7} - \frac{\quad}{\quad} = \frac{55}{7}$

Answer will be between these two factors

Let's find the answer on a number line. We can ask, How many  $\frac{1}{7}$  units are in  $\frac{55}{7}$ . There are 7,  $\frac{1}{7}$ th units in each increment of 1. Let's count and check ( $\frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} = \frac{7}{7}$  or 1).

So we want to multiply  $7 \times (\frac{7}{7})$  which gives us  $\frac{49}{7}$ . How many more  $\frac{1}{7}$ th units do we need to get to 55? That's right, 6 more  $\frac{1}{7}$ th units will get us to  $\frac{55}{7}$  or  $55 \frac{1}{7}$ th units. We can count and check if we want.

Can you see another way to get to  $\frac{55}{7}$  that's easier and faster to find on the number line (hint, look above)?

Right,  $8 \times \frac{7}{7}$  is  $\frac{56}{7}$  so just one more  $\frac{1}{7}$ th unit than we need.

### ***Dimension: Ensure That students Have Adequate Background Knowledge and Skills***

**Activity 1.** Diagnostic assessment is conducted to verify mastery of prerequisite skills. If needed, prerequisite skills are targeted with fluency-building or acquisition instruction.

**Activity 2.** All interventions explicitly connect the targeted skill to skills and understandings that the student already has mastered (e.g., multiplication is taught as repeated addition and mapped onto a number line to make the quantity and operation discriminable).

### ***Dimension: Gradually Fade Support for Correct Execution of Strategies***

**Activity 1.** The teacher models correct responding, provides guided practice with immediate corrective feedback, and then provides an interval of independent practice with delayed error correction in the procedural skill portion of the lesson. The conceptual understanding portion of the lesson is scripted and includes visual supports (number lines), practice creating equivalent quantities, and solving for missing or unknown quantities using modeling and guided practice that requires the student to think aloud, make estimations, and describe observed patterns.

**Activity 2.** Task difficulty is advanced in increments based on student learning gains. For example, if sums to 20 is the terminal skill, sums to 6 and sums to 12 are taught to mastery before introducing sums to 20.

### ***Dimension: Provide Adequate Practice Opportunities***

**Activity 1.** Practice problems are generated to include problems reflecting a particular skill and difficulty level. We generate 5 days of technically equivalent practice problems representing 120% of the mastery criterion for the practice interval plus the follow-up progress monitoring assessment in each weekly intervention packet.

**Activity 2.** All conceptual understanding activities are scripted. Students are typically asked to complete number line activities, solve word problems, complete area models, fill in missing values, create equivalent expressions or quantities, state whether a quantity is greater or lesser, and detect and repair errors.

**Taxonomy Dimension: Behavioral Support**

*Behavioral support addresses the extent to which the program incorporates (a) self-regulation and executive function components and (b) behavioral principles to minimize undesired behavior. Additional information can be found within the [NCII behavioral support course content](#).*

**Activity 1.** Diagnostic assessment is used to verify mastery of prerequisite understandings and determine the need for acquisition support versus fluency-building support such that a student is placed into the correctly aligned intervention skill (task difficulty) and receives the aligned instructional support according to their needs (see Exhibit 4). This makes the learning experience productive, engaging, and rewarding to the student.

**Activity 2.** Weekly progress monitoring and data-driven intervention adjustments ensure that the intervention content and tactic remains well aligned with the student’s needs.

**Activity 3.** Intervention scripts provide the goal to the student (“Remember your score from last time. Your goal today is to beat your score! Remember, your brain is like a muscle. You just worked your math muscle. Now let’s see how much stronger you are getting!”)

**Activity 4.** A self-monitoring chart (Exhibit 10) is completed by each student each day.

**Exhibit 10. Self-Monitoring Chart Example**

<b>Monitor Progress</b>		
Establish Multiplication 0-9		
3/26/2021		
<b>Monitoring Student Progress</b>		
CHART FOR <u>    Muriel Black    </u>		
Weekly Goal: <u>    46    </u>		
DAY 1	My best score is:	_____
	My score on the timed test is:	_____
	Did I beat my score?	_____

**Activity 5.** Daily improvement goals are set, and the student earns rewards each day for performance improvements. In addition, weekly progress monitoring is charted and rewarded with celebrations, small privileges, or tangible rewards for weekly gains.

**Activity 6.** The coach dashboard tracks intervention use and effects throughout the school. Where implementation integrity (dosage and effect) is weak, *SpringMath* alerts the coach to visit the classroom for in-class coaching support to facilitate correct intervention to promote student learning gains and student motivation for continued growth.