Teaching Place-Value Computation (Addition, Subtraction, Multiplication, and Division):
Considerations for Instruction

Purpose and Overview of Guide

The purpose of this guide is to provide strategies and materials for developing and implementing lessons for students who need intensive instruction in the area of computation. Special educators, mathematics interventionists, and others working with students struggling in the area of computation find this guide helpful.

Within college and career ready standards, place value computation is typically taught in grades 1-5. This guide can be used as these concepts are introduced or with students at higher grade levels who continue to struggle with the concepts. Sample activities, worksheets, and supplemental materials also accompany this guide and are available for download at http://www.intensiveintervention.org.

The guide is divided into four sections:

1. Sequence of skills as defined by college and career ready standards.

2. A list of important vocabulary and symbols.

3. A brief explanation of the difficulties students may have with computation.

4. Suggested strategies for teaching computation.

Sequence of Skills – College and Career Ready Standards

(Numbers in parentheses represent the grade level of the standard.)

Add and subtract.

■ Add within 100. (1)

■ Understand that in adding two-digit numbers, add tens and tens, ones and ones. Sometimes it is necessary to compose a ten. (1)

■ Given a two-digit number, mentally find 10 more or 10 less. (1)

■ Subtract multiples of 10 in the range 10-90 from multiple of 10 in the range 10-90. (1)

■ Fluently add and subtract within 100. (2)

- Add up to four two-digit numbers. (2)

- Add and subtract within 1000. (2)

- Understand that in adding or subtracting three-digit numbers, add or subtract hundreds and hundreds, tens and tens, ones and ones. Sometimes it is necessary to compose or decompose tens or hundreds. (2)

- Mentally add or subtract 10 or 100 to or from a given number 100–900. (2)

- Add and subtract within 1000. (3)

- Fluently add and subtract multi-digit whole numbers using the standard algorithm. (4)

- Add and subtract decimals to hundredths. (5)

Multiply and divide.

- Multiply one-digit whole numbers by multiples of 10 in the range 10–90. (3)

- Multiply a whole number of up to four digits by a one-digit whole number. (4)

- Multiply two two-digit numbers. (4)

- Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors. (4)

- Fluently multiply multi-digit whole numbers using the standard algorithm. (5)

- Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors. (5)

- Multiply and divide decimals to hundredths. (5)

Vocabulary and Symbols

The following terms are important for students to understand when working with computation.

Digit: A symbol used to show a number. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9	**Value:** Quantity of a digit. 2 = 2 ones 39 = 3 tens and 9 ones	**Place:** The position of a digit relative to the decimal. Ones, tens, hundreds, etc.
Place Value: The position of a digit relative to the decimal determines the digit's value. 42,103. The value 2 is in the thousands place, so its place value is 2000	**Standard notation:** Writing a number with one digit in each place value. 42,103	**Expanded notation:** Writing a number and showing the place value to each digit. 40,000 + 2,000 + 100 + 3
Word form: Writing a number using words. Forty-two thousand, one-hundred three	**Decimal:** A number written based on powers of ten. 53.109	**Decimal point:** A dot noting the change from positive powers of ten (left of point) to negative powers of ten (right of point). 53.109
Zero: A digit representing the absence of quantity. Zero is necessary in holding place value. 402,005	**Estimate:** An approximate value.	**Round:** An approximate value (usually to the nearest 10, 100, 1000, etc.)
Regroup: To exchange equal amounts of tens and ones, hundreds and tens, thousands and hundreds, etc. 10 ones = 1 ten 1,000 = 10 hundreds	**Trade/exchange/borrow/ carry/rename:** Alternative terms for regrouping.	

Common Areas of Difficulty

Prerequisite skills not mastered:

- Basic fact retrieval

Specific Computation skills:

- *Understanding place value*
  For example, in the number 312, the 1 represents 1 ten, not 1 one.
- *Regrouping*
  Knowing when to regroup 1 hundred into 10 tens.
  Notation for computation with regrouping.
- *Estimating*
  How to round to the nearest designated power of 10.

Developing Conceptual Understanding

Base-10 blocks can be used to help students understand the concepts behind whole number computation. Base-10 blocks can also be used to explain decimals. Other place value manipulatives include: Unifix cubes, snap cubes, plastic clips, and beans sticks/beans.

Activities and Strategies Related to Specific Standards

Addition of Whole Numbers

- Cubes (1000), flats (100), rods (10), and units (1) are used for addition with and without regrouping.
- It is helpful to have a place value mat so students can organize the blocks.
- Represent each addend with blocks and add the two (or more) addends together.

- For example:
  - A student is presented with the problem:
  - Say, "Let's represent the first addend, 148."
  - Place 1 flat, 4 rods, and 8 units on the mat.
  - Say, "Let's add 75."
  - Place 7 rods and 5 units on the mat. (Notice how 75 is presented below 148. The set-up with blocks mirrors the set-up with numbers.)
  - Say, "When adding, we start in the ones column with the units. Let's add all the units. We have 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 units. We can't have more than 9 units in the ones column, so we regroup 10 units for 1 rod."
  - Say, "Now we move to the tens column with the rods. Let's add all the rods. We have 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 rods. We can't have more than 9 rods in the tens column, so we regroup 10 rods for 1 flat."
  - Say, "Now we move to the hundreds column with the flats. Let's add all the flats. We have 1, 2 flats."
  - Confirm "148 plus 75 equals 223."
- Explain the notation (writing the regrouping) as soon as students understand the concept of regrouping.

$$\begin{array}{r} 1\ 4\ 8 \\ +\ 7\ 5 \\ \hline \end{array}$$

Hundreds	Tens	Ones
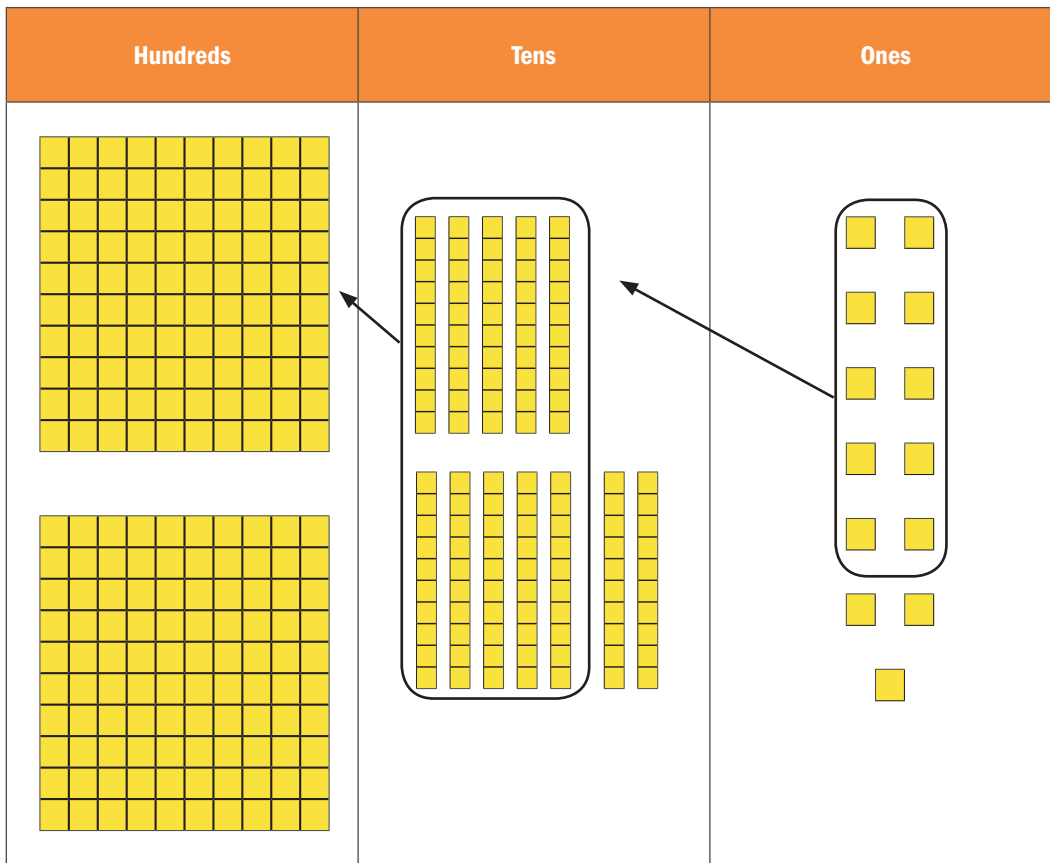
Addition with paper-and-pencil

- Teach students to line up numbers according to place value. (Graph paper or notebook paper turned sideways may help. Also, you can draw lines between place value columns.)

- Using the standard algorithm:

```
    1
  4 1 7
+ 2 6 8
  6 8 5
```

  - Add the ones.
  - If the answer is more than 9, regroup.
  - Write ones answer.
  - Add the tens.
  - If the answer is more than 9, regroup.
  - Write tens answer.
  - Add the hundreds.
  - If the answer is more than 9, regroup.
  - Write hundreds answer.

- Using the partial sums algorithm:

```
    4 1 7
+   2 6 8
  ─────────
    6 0 0
      7 0
      1 5
  ─────────
    6 8 5
```

  - Add the hundreds.
  - Write hundreds answer (as a hundred).
  - Add the tens.
  - Write tens answer (as a ten).
  - Add the ones.
  - Write ones answer.
  - Add partial sums.

Subtraction of Whole Numbers

- Cubes (1000), flats (100), rods (10), and units (1) are used for subtraction with and without regrouping.

- It is helpful to have a place value mat so students can organize the blocks.

- Represent the minuend and take away the subtrahend from the minuend. Do not represent the subtrahend with blocks.

- For example:

```
    2 3 4
  − 1 8 5
  ─────────
```

  - A student is presented with the problem:
  - Say, "Let's represent the minuend, 234."
  - Place 2 flats, 3 rods, and 4 units on the mat.
  - Say, "Let's subtract 185."

- Say, "When subtracting, we start in the ones column with the units. Let's subtract 5 units. But wait, we do not have enough units to take away 5. So, we have to regroup. We take 1 rod from the tens column and regroup the rod into 10 units. Now we can take away 5 units." Take away (or cross out) 1, 2, 3, 4, 5 units.

- Say, "Now we move to the tens column with the rods. Let's subtract 8 rods. But wait, we do not have enough rods to take away 8. So, we have to regroup. We take 1 flat from the hundreds column and regroup the flat into 10 rods. Now we can take away 8 rods." Take away (or cross out) 1, 2, 3, 4, 5, 6, 7, 8 rods.

- Say, "Now we move to the hundreds column with the flats. Let's subtract 1 flat." Take away (or cross out) 1 flat.

- Confirm "234 – 185 = 49."

■ Explain the notation (writing the regrouping) as soon as students understand the concept of regrouping.

Hundreds	Tens	Ones
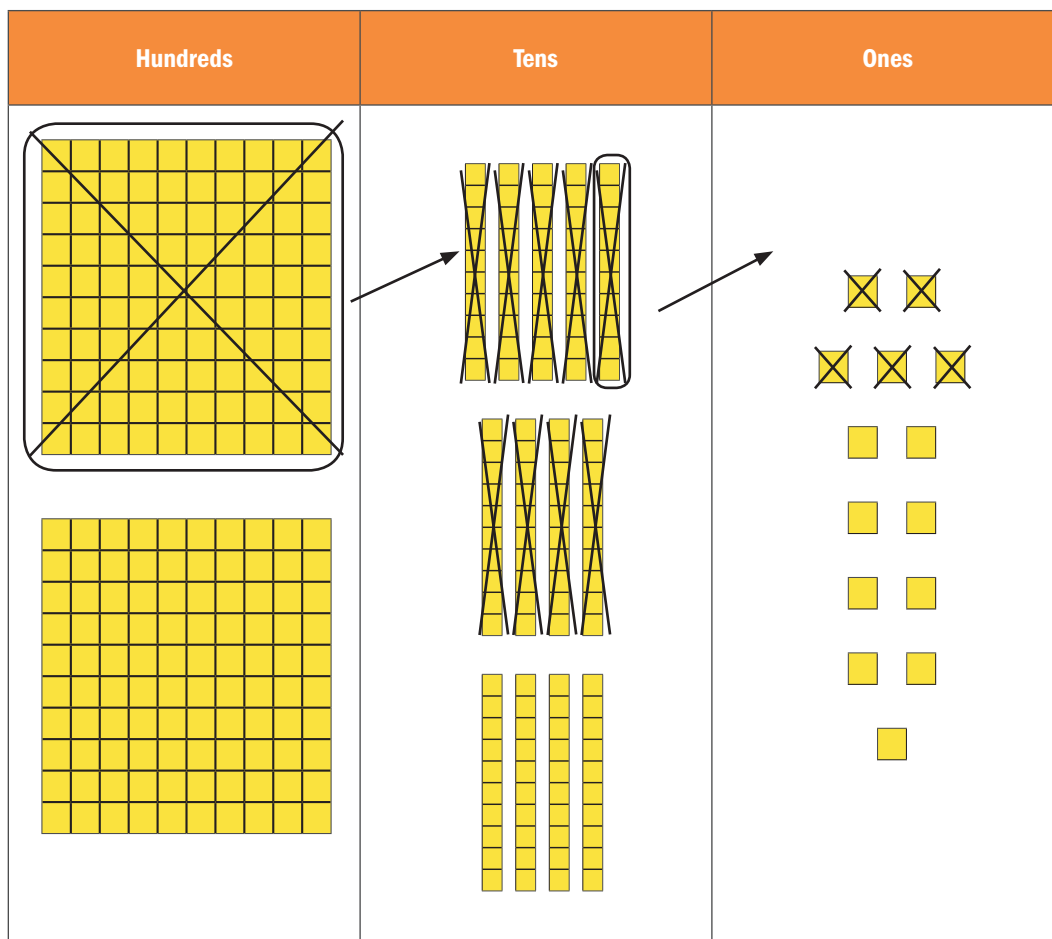
Subtraction with paper-and-pencil

- Teach students to line up numbers according to place value.
- ■ Using the standard algorithm: ————————————→

	5	¹0	
	$\cancel{6}$	$\cancel{1}$	¹4
−	2	5	7
	3	5	7

  - Subtract the ones.
  - If there aren't enough ones to subtract, regroup.
  - (To regroup, take 1 ten and break into 10 ones.)
  - Write ones answer.
  - Subtract the tens.
  - If there aren't enough tens to subtract, regroup.
  - (To regroup, take 1 hundred and break into 10 tens.)
  - Write tens answer.
  - Subtract the hundreds.
  - Write hundreds answer.

- ■ Using the partial differences algorithm: ————————→

  ```
        6 1 4
    −   2 5 7
    _____
    +   4 0 0
    −     4 0
    −       3
    _____
        3 5 7
  ```

  - Subtract the hundreds.
  - (If the bottom number is smaller, use a + sign.)
  - (If the bottom number is bigger, use a – sign.)
  - Write hundreds answer.
  - Subtract the tens.
  - Write tens answer.
  - Subtract the ones.
  - Write ones answer.
  - Compute the partial differences.

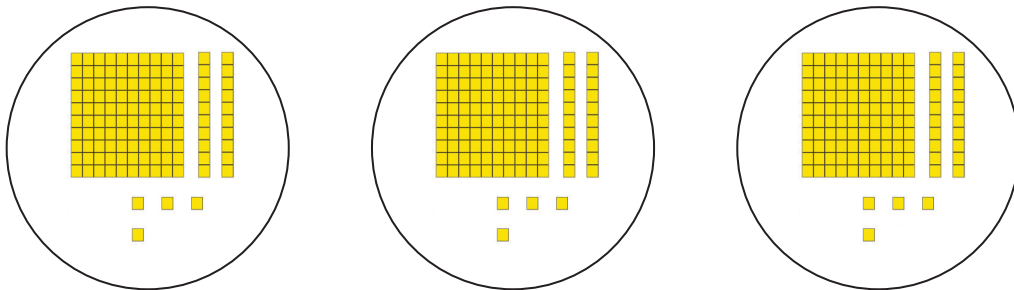## Multiplication of Whole Numbers

- ■ Cubes (1000), flats (100), rods (10), and units (1) are used for multiplication.
- ■ It is helpful to have a material (like paper plates or cups) to denote the groups.
- ■ Only use Base-10 blocks for multiplication with smaller numbers.
- ■ For example:

  ```
      1 2 4
  x       3
  _____
  ```

  - A student is presented with the problem: ————————→
  - Say, "We have 3 groups of 124."
  - Say, "Let's first show 3 groups by placing 3 plates on the table."
  - Place 3 plates.
  - Say, "Now, each plate, or group, has 124. Let's place 1 flat, 2 rods, and 4 units on each plate."

- Place 1 flat, 2 rods, and 4 units on each plate.
- Say, "When adding, we start in the ones column with the units. Multiplication is repeated addition, so let's add all the units. We have 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 units. We can't have more than 9 units in the ones column, so we regroup 10 units for 1 rod."
- Say, "Now we move to the tens column with the rods. Let's add all the rods. We have 1, 2, 3, 4, 5, 6, 7 rods."
- Say, "Now we move to the hundreds column with the flats. Let's add all the flats. We have 1, 2, 3 flats."
- Confrim "124 times 3 equals 372."

■ Explain the notation (writing the regrouping) as soon as students understand the concept of regrouping.

Multiplication with paper-and-pencil

- Teach students to line up numbers according to place value.

■ Using the standard algorithm:
  - Multiply bottom one by top one.
  - Write answer. (Regroup if necessary.)
  - Multiply bottom one by top ten.
  - Add regrouped tens, if necessary.
  - Write answer. (Regroup if necessary.)
  - Bring down regrouped hundreds.
  - Cross out all ones regroupings.
  - Multiply bottom ten by top one.
  - Write answer. (Regroup if necessary.)
  - Multiply bottom ten by top ten.
  - Add regrouped hundreds, if necessary.
 - Write answer. (Regroup if necessary).
  - Add.

- Using the partial products algorithm:
  - Multiply top ten by bottom ten.
  - Write answer.
  - Multiply top ten by bottom one.
  - Write answer.
  - Multiply top one by bottom ten.
  - Write answer.
  - Multiply top one by bottom one.
  - Write answer.
  - Add partial products.

```
        2 3
  x     1 9
  ---------
      2 0 0
      1 8 0
        3 0
  +     2 7
  ---------
      4 3 7
```

- Using the lattice method:
  - Draw lattice.
  - Write first factor on top.
  - Write second factor to right.
  - Multiply top ten by side ten.
  - Write answer. (Tens above lattice, ones below.)
  - Multiply top one by side ten.
  - Write answer.
  - Multiply top ten by side one.
  - Write answer.
  - Multiply top one by side one.
  - Write answer.
  - Treat the lattices as thousands, hundreds, tens, and ones.
  - Add lattices. (Regroup if necessary.)
  - Answer starts at top of left side and works around bottom corner: 437.
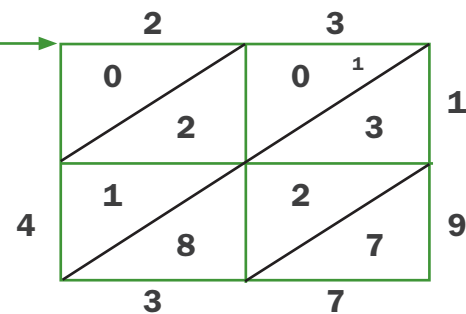
Division of Whole Numbers

- Cubes (1000), flats (100), rods (10), and units (1) are used for division.
- It is helpful to have a material (like paper plates or cups) to denote the groups.
- Only use Base-10 blocks for division with smaller numbers.
- For example:
  - A student is presented with the problem:
  - Say, "We have to divide 164 into 3 groups."
  - Say, "Let's first show 164. That's 1 flat, 6 rods, and 4 units."
  - Place 1 flat, 6 rods, and 4 units.

- Say, "Now, we have 3 plates, or groups. We start in the hundreds. Can we divide 1 flat evenly? No. We have to regroup 1 flat into 10 rods."
- Regroup 1 flat for 10 rods.
- Say, "Now we have 16 rods. Let's divide the rods evenly into the 3 groups."
- Place 5 rods on each plate.
- Say, "We have 1 rod leftover. To divide the rod, we need to break it into 10 units."
- Regroup 1 rod for 10 units.
- Say, "Now we have 14 units. Let's divide the units evenly into the 3 groups."
- Place 4 units on each plate.
- Say, "We have 2 units left over. 2 is our remainder."
- Confirm "164 divided by 3 equals 54 remainder of 2."

■ Explain the notation (writing the regrouping) as soon as students understand the concept of regrouping.

Division with paper-and-pencil

- Teach students to line up numbers according to place value.

■ Using the standard algorithm: ────────────▶

- Divide the hundreds by the divisor.
- Write answer above hundreds.
- Multiply divisor by hundreds answer.
- Write product below hundreds.
- Subtract.
- Bring down tens.
- Divide the difference by the divisor.
- Write answer above tens.
- Multiply divisor by tens answer.
- Write product below.
- Subtract.
- Bring down ones.
- Divide the difference by the divisor.
- Write answer above ones.
- Multiply divisor by ones answer.
- Write product below.
- Subtract.
- Determine if there is a remainder.

$$
\begin{array}{r}
144\ \text{R3} \\
5\overline{)723} \\
-5 \\
\hline
22 \\
-20 \\
\hline
23 \\
-20 \\
\hline
3
\end{array}
$$

- Using the partial quotients algorithm:
  - Draw vertical line from end of the division line.
  - Determine (about) how many times 5 goes into the number representing the hundreds.
  - Write answer to right of vertical line.
  - Multiply. (Estimate × divisor.)
  - Subtract.
  - Determine (about) how many times 5 goes into difference
  - (200).
  - Write answer to right of vertical line.
  - Multiply. (Estimate × divisor.)
  - Subtract.
  - Determine (about) how many times 5 goes into difference (20).
  - Write answer to right of vertical line.
  - Multiply. (Estimate × divisor.)
  - Subtract.
  - Determine (about) how many times 5 goes into ones.
  - Write answer to right of vertical line.
  - Multiply. (Estimate × divisor.)
  - Subtract.
  - If this difference is not 0, the difference is the remainder.

$$
\begin{array}{r|l}
5\overline{)\,7\ 2\ 3} & \text{reds} \\
-5\ 0\ 0 & 100 \\ \hline
2\ 0\ 0 & 40 \\
-\ 2\ 0\ 0 & \\ \hline
2\ 0 & 4 \\
-\ 2\ 0 & \\ \hline
3 & \\
& 144R3
\end{array}
$$

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