National Center on
INTENSIVE INTERVENTION
at American Institutes for Research ■

Place-Value Computation

Contents

Teaching Place-Value Computation (Addition, Subtraction, Multiplication, and Division):
Considerations for Instruction

Purpose and Overview of Guide

The purpose of this guide is to provide strategies and materials for developing and implementing lessons for students who need intensive instruction in the area of computation. Special educators, mathematics interventionists, and others working with students struggling in the area of computation find this guide helpful.

Within college and career ready standards, place value computation is typically taught in grades 1-5. This guide can be used as these concepts are introduced or with students at higher grade levels who continue to struggle with the concepts. Sample activities, worksheets, and supplemental materials also accompany this guide and are available for download at http://www.intensiveintervention.org.

The guide is divided into four sections:

1. Sequence of skills as defined by college and career ready standards.

2. A list of important vocabulary and symbols.

3. A brief explanation of the difficulties students may have with computation.

4. Suggested strategies for teaching computation.

Sequence of Skills – College and Career Ready Standards

(Numbers in parentheses represent the grade level of the standard.)

Add and subtract.

■ Add within 100. (1)

■ Understand that in adding two-digit numbers, add tens and tens, ones and ones. Sometimes it is necessary to compose a ten. (1)

■ Given a two-digit number, mentally find 10 more or 10 less. (1)

■ Subtract multiples of 10 in the range 10-90 from multiple of 10 in the range 10-90. (1)

■ Fluently add and subtract within 100. (2)

- Add up to four two-digit numbers. (2)

- Add and subtract within 1000. (2)

- Understand that in adding or subtracting three-digit numbers, add or subtract hundreds and hundreds, tens and tens, ones and ones. Sometimes it is necessary to compose or decompose tens or hundreds. (2)

- Mentally add or subtract 10 or 100 to or from a given number 100–900. (2)

- Add and subtract within 1000. (3)

- Fluently add and subtract multi-digit whole numbers using the standard algorithm. (4)

- Add and subtract decimals to hundredths. (5)

Multiply and divide.

- Multiply one-digit whole numbers by multiples of 10 in the range 10–90. (3)

- Multiply a whole number of up to four digits by a one-digit whole number. (4)

- Multiply two two-digit numbers. (4)

- Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors. (4)

- Fluently multiply multi-digit whole numbers using the standard algorithm. (5)

- Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors. (5)

- Multiply and divide decimals to hundredths. (5)

Vocabulary and Symbols

The following terms are important for students to understand when working with computation.

Digit: A symbol used to show a number. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9	Value: Quantity of a digit. 2 = 2 ones 39 = 3 tens and 9 ones	Place: The position of a digit relative to the decimal. Ones, tens, hundreds, etc.
Place Value: The position of a digit relative to the decimal determines the digit's value. 42,103. The value 2 is in the thousands place, so its place value is 2000	Standard notation: Writing a number with one digit in each place value. 42,103	Expanded notation: Writing a number and showing the place value to each digit. 40,000 + 2,000 + 100 + 3
Word form: Writing a number using words. Forty-two thousand, one-hundred three	Decimal: A number written based on powers of ten. 53.109	Decimal point: A dot noting the change from positive powers of ten (left of point) to negative powers of ten (right of point). 53.109
Zero: A digit representing the absence of quantity. Zero is necessary in holding place value. 402,005	Estimate: An approximate value.	Round: An approximate value (usually to the nearest 10, 100, 1000, etc.)
Regroup: To exchange equal amounts of tens and ones, hundreds and tens, thousands and hundreds, etc. 10 ones = 1 ten 1,000 = 10 hundreds	Trade/exchange/borrow/carry/rename: Alternative terms for regrouping.	

Common Areas of Difficulty

Prerequisite skills not mastered:

- Basic fact retrieval

Specific Computation skills:

- *Understanding place value*
  For example, in the number 312, the 1 represents 1 ten, not 1 one.

- *Regrouping*
  Knowing when to regroup 1 hundred into 10 tens.
  Notation for computation with regrouping.

- *Estimating*
  How to round to the nearest designated power of 10.

Developing Conceptual Understanding

Base-10 blocks can be used to help students understand the concepts behind whole number computation. Base-10 blocks can also be used to explain decimals. Other place value manipulatives include: Unifix cubes, snap cubes, plastic clips, and beans sticks/beans.

Activities and Strategies Related to Specific Standards

Addition of Whole Numbers

- Cubes (1000), flats (100), rods (10), and units (1) are used for addition with and without regrouping.

- It is helpful to have a place value mat so students can organize the blocks.

- Represent each addend with blocks and add the two (or more) addends together.

- For example:
  - A student is presented with the problem:

$$\begin{array}{r} 1\ 4\ 8 \\ +\ 7\ 5 \\ \hline \end{array}$$

  - Say, "Let's represent the first addend, 148."
  - Place 1 flat, 4 rods, and 8 units on the mat.
  - Say, "Let's add 75."
  - Place 7 rods and 5 units on the mat. (Notice how 75 is presented below 148. The set-up with blocks mirrors the set-up with numbers.)
  - Say, "When adding, we start in the ones column with the units. Let's add all the units. We have 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 units. We can't have more than 9 units in the ones column, so we regroup 10 units for 1 rod."
  - Say, "Now we move to the tens column with the rods. Let's add all the rods. We have 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 rods. We can't have more than 9 rods in the tens column, so we regroup 10 rods for 1 flat."
  - Say, "Now we move to the hundreds column with the flats. Let's add all the flats. We have 1, 2 flats."
  - Confirm "148 plus 75 equals 223."
- Explain the notation (writing the regrouping) as soon as students understand the concept of regrouping.

Hundreds	Tens	Ones
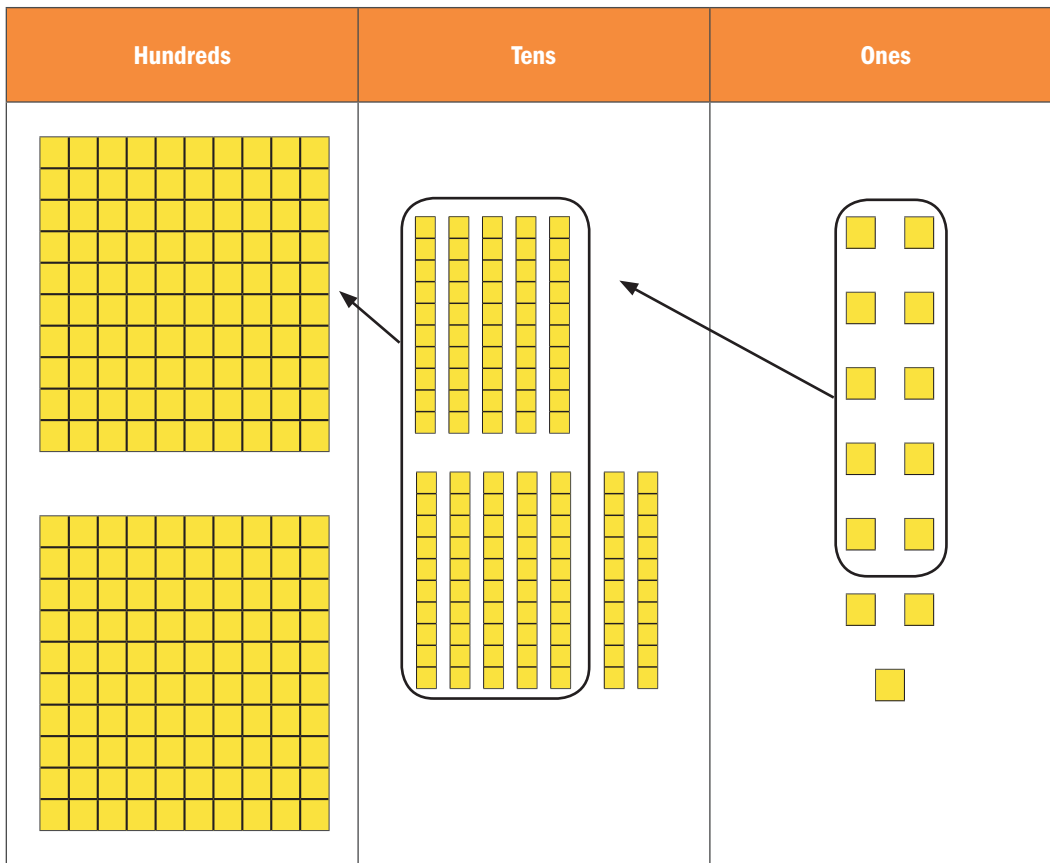
Addition with paper-and-pencil

- Teach students to line up numbers according to place value. (Graph paper or notebook paper turned sideways may help. Also, you can draw lines between place value columns.)

- Using the standard algorithm:

  - Add the ones.
  - If the answer is more than 9, regroup.
  - Write ones answer.
  - Add the tens.
  - If the answer is more than 9, regroup.
  - Write tens answer.
  - Add the hundreds.
  - If the answer is more than 9, regroup.
  - Write hundreds answer.

	1	
4	1	7
+ 2	6	8
6	8	5

- Using the partial sums algorithm:

  - Add the hundreds.
  - Write hundreds answer (as a hundred).
  - Add the tens.
  - Write tens answer (as a ten).
  - Add the ones.
  - Write ones answer.
  - Add partial sums.

```
    4 1 7
+   2 6 8
_____
    6 0 0
      7 0
      1 5
_____
    6 8 5
```

Subtraction of Whole Numbers

- Cubes (1000), flats (100), rods (10), and units (1) are used for subtraction with and without regrouping.

- It is helpful to have a place value mat so students can organize the blocks.

- Represent the minuend and take away the subtrahend from the minuend. Do not represent the subtrahend with blocks.

- For example:

  - A student is presented with the problem:
  - Say, "Let's represent the minuend, 234."
  - Place 2 flats, 3 rods, and 4 units on the mat.
  - Say, "Let's subtract 185."

```
    2 3 4
−   1 8 5
```

- Say, "When subtracting, we start in the ones column with the units. Let's subtract 5 units. But wait, we do not have enough units to take away 5. So, we have to regroup. We take 1 rod from the tens column and regroup the rod into 10 units. Now we can take away 5 units." Take away (or cross out) 1, 2, 3, 4, 5 units.

- Say, "Now we move to the tens column with the rods. Let's subtract 8 rods. But wait, we do not have enough rods to take away 8. So, we have to regroup. We take 1 flat from the hundreds column and regroup the flat into 10 rods. Now we can take away 8 rods." Take away (or cross out) 1, 2, 3, 4, 5, 6, 7, 8 rods.

- Say, "Now we move to the hundreds column with the flats. Let's subtract 1 flat." Take away (or cross out) 1 flat.

- Confirm "234 – 185 = 49."

■ Explain the notation (writing the regrouping) as soon as students understand the concept of regrouping.

Hundreds	Tens	Ones
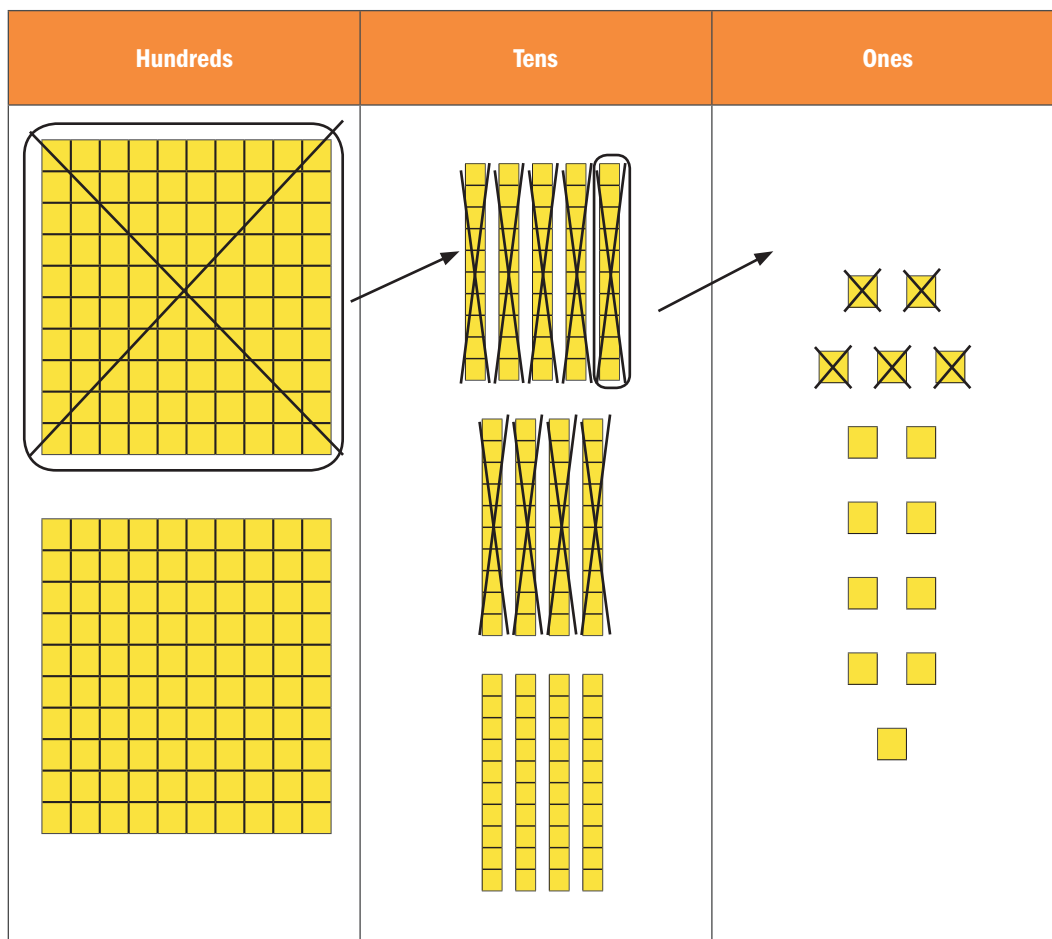
Subtraction with paper-and-pencil

- Teach students to line up numbers according to place value.

- Using the standard algorithm:

  - Subtract the ones.
  - If there aren't enough ones to subtract, regroup.
  - (To regroup, take 1 ten and break into 10 ones.)
  - Write ones answer.
  - Subtract the tens.
  - If there aren't enough tens to subtract, regroup.
  - (To regroup, take 1 hundred and break into 10 tens.)
  - Write tens answer.
  - Subtract the hundreds.
  - Write hundreds answer.

$$
\begin{array}{r}
5 \; {}^{1}0 \\
\cancel{6} \; \cancel{1} \; {}^{1}4 \\
- \; 2 \; 5 \; 7 \\
\hline
3 \; 5 \; 7
\end{array}
$$

- Using the partial differences algorithm:

  - Subtract the hundreds.
  - (If the bottom number is smaller, use a + sign.)
  - (If the bottom number is bigger, use a – sign.)
  - Write hundreds answer.
  - Subtract the tens.
  - Write tens answer.
  - Subtract the ones.
  - Write ones answer.
  - Compute the partial differences.

$$
\begin{array}{r}
6 \; 1 \; 4 \\
- \quad 2 \; 5 \; 7 \\
\hline
+ \quad 4 \; 0 \; 0 \\
- \quad \; 4 \; 0 \\
- \quad \; 3 \\
\hline
3 \; 5 \; 7
\end{array}
$$

Multiplication of Whole Numbers

- Cubes (1000), flats (100), rods (10), and units (1) are used for multiplication.

- It is helpful to have a material (like paper plates or cups) to denote the groups.

- Only use Base-10 blocks for multiplication with smaller numbers.

- For example:

  - A student is presented with the problem:
  - Say, "We have 3 groups of 124."
  - Say, "Let's first show 3 groups by placing 3 plates on the table."
  - Place 3 plates.
  - Say, "Now, each plate, or group, has 124. Let's place 1 flat, 2 rods, and 4 units on each plate."

$$
\begin{array}{r}
1 \; 2 \; 4 \\
\times \quad \; 3 \\
\hline
\end{array}
$$

- Place 1 flat, 2 rods, and 4 units on each plate.
- Say, "When adding, we start in the ones column with the units. Multiplication is repeated addition, so let's add all the units. We have 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 units. We can't have more than 9 units in the ones column, so we regroup 10 units for 1 rod."
- Say, "Now we move to the tens column with the rods. Let's add all the rods. We have 1, 2, 3, 4, 5, 6, 7 rods."
- Say, "Now we move to the hundreds column with the flats. Let's add all the flats. We have 1, 2, 3 flats."
- Confrim "124 times 3 equals 372."
■ Explain the notation (writing the regrouping) as soon as students understand the concept of regrouping.
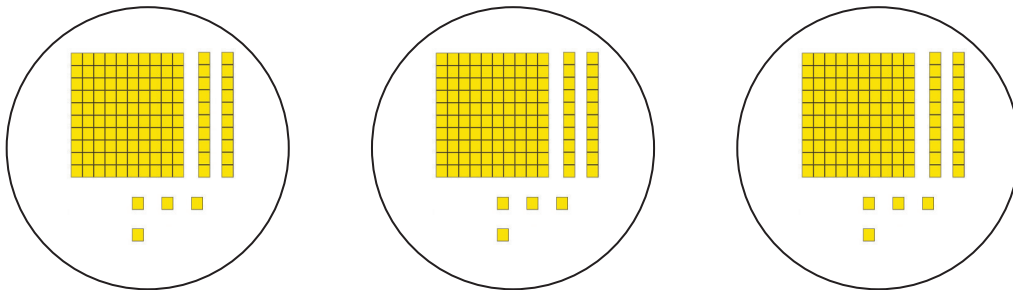
Multiplication with paper-and-pencil
- Teach students to line up numbers according to place value.

■ Using the standard algorithm:
- Multiply bottom one by top one.
- Write answer. (Regroup if necessary.)
- Multiply bottom one by top ten.
- Add regrouped tens, if necessary.
- Write answer. (Regroup if necessary.)
- Bring down regrouped hundreds.
- Cross out all ones regroupings.
- Multiply bottom ten by top one.
- Write answer. (Regroup if necessary.)
- Multiply bottom ten by top ten.
- Add regrouped hundreds, if necessary.
- Write answer. (Regroup if necessary).
- Add.

- Using the partial products algorithm:
  - Multiply top ten by bottom ten.
  - Write answer.
  - Multiply top ten by bottom one.
  - Write answer.
  - Multiply top one by bottom ten.
  - Write answer.
  - Multiply top one by bottom one.
  - Write answer.
  - Add partial products.

```
          2 3
   x      1 9
   _____
        2 0 0
        1 8 0
          3 0
   +      2 7
   _____
        4 3 7
```

- Using the lattice method:
  - Draw lattice.
  - Write first factor on top.
  - Write second factor to right.
  - Multiply top ten by side ten.
  - Write answer. (Tens above lattice, ones below.)
  - Multiply top one by side ten.
  - Write answer.
  - Multiply top ten by side one.
  - Write answer.
  - Multiply top one by side one.
  - Write answer.
  - Treat the lattices as thousands, hundreds, tens, and ones.
  - Add lattices. (Regroup if necessary.)
  - Answer starts at top of left side and works around bottom corner: 437.
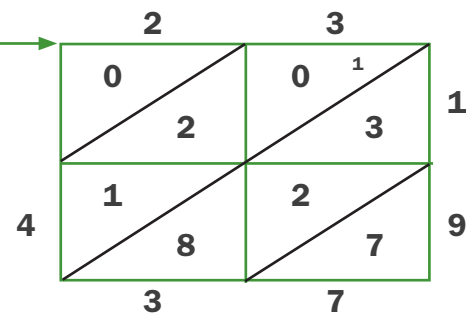
Division of Whole Numbers

- Cubes (1000), flats (100), rods (10), and units (1) are used for division.
- It is helpful to have a material (like paper plates or cups) to denote the groups.
- Only use Base-10 blocks for division with smaller numbers.
- For example:
  - A student is presented with the problem:
  - Say, "We have to divide 164 into 3 groups."
  - Say, "Let's first show 164. That's 1 flat, 6 rods, and 4 units."
  - Place 1 flat, 6 rods, and 4 units.

- Say, "Now, we have 3 plates, or groups. We start in the hundreds. Can we divide 1 flat evenly? No. We have to regroup 1 flat into 10 rods."
- Regroup 1 flat for 10 rods.
- Say, "Now we have 16 rods. Let's divide the rods evenly into the 3 groups."
- Place 5 rods on each plate.
- Say, "We have 1 rod leftover. To divide the rod, we need to break it into 10 units."
- Regroup 1 rod for 10 units.
- Say, "Now we have 14 units. Let's divide the units evenly into the 3 groups."
- Place 4 units on each plate.
- Say, "We have 2 units left over. 2 is our remainder."
- Confirm "164 divided by 3 equals 54 remainder of 2."

■ Explain the notation (writing the regrouping) as soon as students understand the concept of regrouping.

Division with paper-and-pencil

- Teach students to line up numbers according to place value.

■ Using the standard algorithm:
- Divide the hundreds by the divisor.
- Write answer above hundreds.
- Multiply divisor by hundreds answer.
- Write product below hundreds.
- Subtract.
- Bring down tens.
- Divide the difference by the divisor.
- Write answer above tens.
- Multiply divisor by tens answer.
- Write product below.
- Subtract.
- Bring down ones.
- Divide the difference by the divisor.
- Write answer above ones.
- Multiply divisor by ones answer.
- Write product below.
- Subtract.
- Determine if there is a remainder.

$$
\begin{array}{r}
144 \text{ R3} \\
5 \overline{)\,723} \\
-5 \\
\hline
22 \\
-20 \\
\hline
23 \\
-20 \\
\hline
3
\end{array}
$$

- Using the partial quotients algorithm:
  - Draw vertical line from end of the division line.
  - Determine (about) how many times 5 goes into the number representing the hundreds.
  - Write answer to right of vertical line.
  - Multiply. (Estimate × divisor.)
  - Subtract.
  - Determine (about) how many times 5 goes into difference
  - (200).
  - Write answer to right of vertical line.
  - Multiply. (Estimate × divisor.)
  - Subtract.
  - Determine (about) how many times 5 goes into difference (20).
  - Write answer to right of vertical line.
  - Multiply. (Estimate × divisor.)
  - Subtract.
  - Determine (about) how many times 5 goes into ones.
  - Write answer to right of vertical line.
  - Multiply. (Estimate × divisor.)
  - Subtract.
  - If this difference is not 0, the difference is the remainder.

$$
\begin{array}{r}
5 \overline{)\ 7\ 2\ 3} \\
-\ 5\ 0\ 0 \\
\hline
2\ 0\ 0 \\
-\ 2\ 0\ 0 \\
\hline
2\ 0 \\
-\ 2\ 0 \\
\hline
3 \\
\end{array}
$$

reds
100
40
4

144R3

2. Use Place-Value Understanding and Properties of Addition and Subtraction to Add and Subtract

Sample Activities
- a) Activity One: Using Base-10 Blocks for Addition
- b) Activity Two: Using Base-10 Blocks for Subtraction

Worksheets
- a) Place-Value Computation: Addition
- b) Place-Value Computation: Subtraction

Sample Computation Concept:

Use Place-Value Understanding and Properties of Addition and Subtraction to Add and Subtract

College and Career Ready Standards Addressed

2.NBT.7 Add and subtract within 1,000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

Activity One: Using Base-10 Blocks for Addition

Purpose:

To relate concrete objects to a written method for adding and subtracting 3-digit numbers

Principles of Intensive Intervention Illustrated:

- Provide concrete learning opportunities (including use of manipulatives).
- Provide explicit error correction, and have students repeat the correct process.
- Use precise, simple language to teach key concepts or procedures.
- Use explicit instruction and modeling with repetition to teach a concept or demonstrate steps in a process.
- Provide repeated opportunities to practice each step correctly.

Materials:

- Base-10 blocks (see Supplemental Materials)
- Place-value mat with columns for hundreds (flats), tens (rods), ones (units) (see Supplemental Materials)
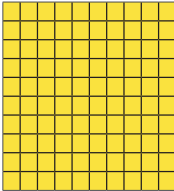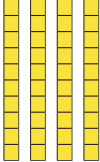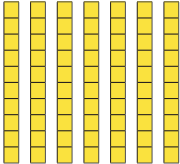
Hundreds	Tens	Ones

- Paper and pencil or dry-erase board
- Worksheet: Place-Value Computation: Addition (for extra practice)
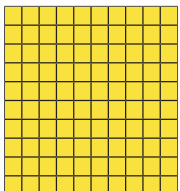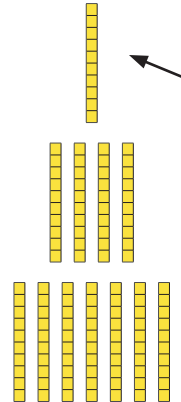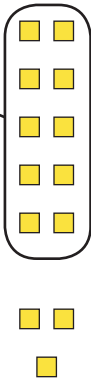
Modeling:

1. Explain to the student that base-10 blocks can be used to demonstrate addition of multidigit numbers.

2. Write a multidigit addition problem (e.g., 148 + 75). Use graph paper or notebook paper turned sideways to line up place values of the addends.

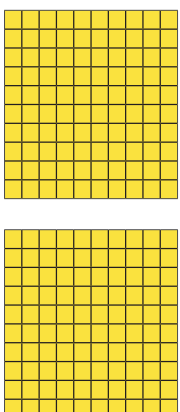|	1	4	8
+		7	5
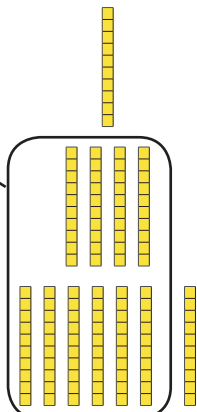
3. Represent each addend using base-10 blocks on the place-value mat.

Hundreds	Tens	Ones
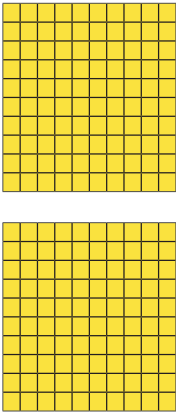
4. Start in the ones column of the place-value mat and add together all ones (units). If greater than 9, exchange 10 ones for 1 ten (rod) and put the rod in the tens column. Refer to the written problem. Add 8 + 5 in the ones column, write 3 below the equal sign (horizontal line) and write the digit 1 above the tens column. Make it clear to the student by pointing to the written problem and the objects on the place-value mat and show how they correspond.

Hundreds	Tens	Ones

5. Move to the tens column on the place-value mat. Add together all tens (rods), including the extra ten added by exchanging the 10 ones. If more than 9, exchange for 1 hundred (flat) and place it in the hundreds column. Refer to the written problem. Add 4 + 7 + 1 in the tens column, write 2 below the horizontal line and write the digit 1 above the hundreds column. Make it clear to the student how the written problem and place-value mat correspond.

Hundreds	Tens	Ones

6. Move to the hundreds column on the place-value mat. Add together all hundreds (flats), including the extra hundred added by exchanging the 10 tens. Refer to the written problem. Add 1 +1 in hundreds column, write 2 below the horizontal line. Make it clear to the student how the mat and the written problem correspond.

Hundreds	Tens	Ones

7. Have the student read the completed problem:

   148 + 75 = 223.

   Provide <u>at least</u> two more examples (e.g., 216 + 50; 309 + 111).

	1	1	
	1	4	8
+		7	5
	2	2	3

Guided Practice:

1. Have the student write down the multidigit addition problem (e.g., 318 + 56). Use graph or lined paper, if needed, to maintain place-value columns.

2. Student represents both addends on the place-value mat.

3. Beginning with the ones column, student adds all ones (units), exchanging for tens (rods) if possible. Student completes the ones column of the written problem.

4. Student repeats with the tens column, then the hundreds column.

5. Student reads the completed answer.

6. Repeat with two more examples (e.g., 602 + 205; 472 + 328).

Corrective Feedback:

Example 1

Student response: The student adds 8 + 5 in the ones column and writes 13 below the horizontal line.

Teacher feedback: Use graph paper so that only 1 digit can be written in each place-value column. Or, have the student estimate the sum before adding and then have him or her judge the reasonableness of the answer. Have the student demonstrate the correct process before moving on.

Activity Two: Using Base-10 Blocks for Subtraction

Purpose:

To relate concrete objects to a written method for adding and subtracting 3-digit numbers

Principles of Intensive Intervention Illustrated:

- Provide concrete learning opportunities (including use of manipulatives).
- Provide explicit error correction, and have students repeat the correct process.
- Use precise, simple language to teach key concepts or procedures.
- Use explicit instruction and modeling with repetition to teach a concept or demonstrate steps in a process.
- Provide repeated opportunities to practice each step correctly.

Materials:
- Base-10 blocks (see Supplemental Materials)
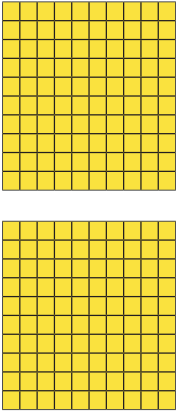- Place-value mat with columns for hundreds (flats), tens (rods), ones (units) (see Supplemental Materials)

- Paper and pencil or dry-erase board
- Worksheet: Place-Value Computation: Subtraction (for extra practice)

Modeling:
1. Explain to the student that base-10 blocks also can be used to demonstrate subtraction of multidigit numbers.

2. Write a multidigit subtraction problem (e.g., 234 – 88). Use graph paper or notebook paper turned sideways to line up place values of minuend (starting amount) and subtrahend (amount begin subtracted).

3. Represent the minuend (234) with base-10 blocks on the place-value mat (4 ones = 4 units, 3 tens = 3 rods, 2 hundreds = 2 flats).

Hundreds	Tens	Ones

4. Refer to the written problem. Remind the student to start in the ones column when subtracting. In this example, it is not possible to subtract 8 ones from 4 ones; it's necessary to exchange 1 ten (rod) for 10 ones (units). Demonstrate for the student with manipulatives on the place-value mat. After exchanging 1 ten for 10 ones, there are now 2 tens instead of 3 tens and there are 14 ones instead of 4. It is now possible to subtract 8 ones and have 6 ones remaining. Model the steps on the written problem: Cross out the 3 in the minuend and write a 2, then write a 1 in front of the 4 to show 14 ones. Then, write a 6 below the equal sign (horizontal line) in the ones column to show there are 6 ones remaining.

Hundreds	Tens	Ones
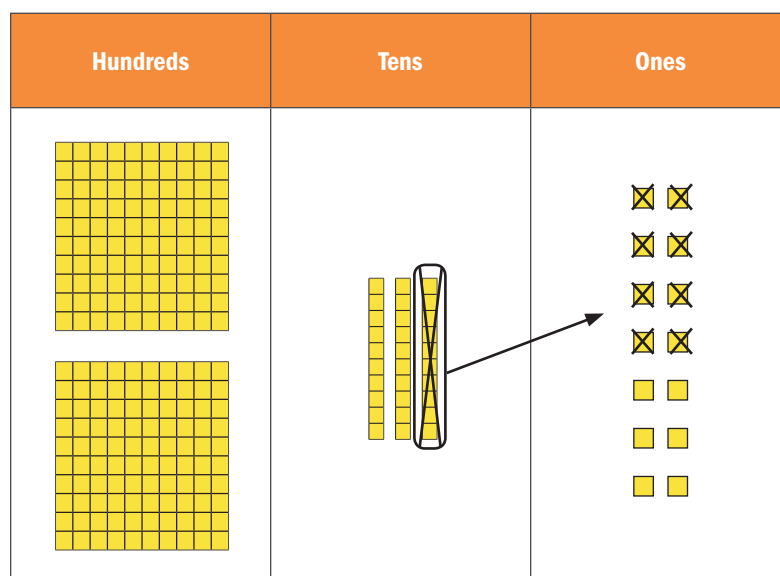
5. Refer to the written problem. In the tens column, it is not possible to subtract 8 tens from 2 tens. Exchanging 1 hundred (flat) for 10 tens (rods) is necessary. Demonstrate for the student with manipulatives on the place-value mat. After exchanging 1 hundred for 10 tens, there is 1 hundred (flat) remaining instead of 2 hundreds, and there are 12 tens (rods) instead of 2 tens. Subtract 8 tens (rods) from 12 tens and have 4 tens remaining. Model on the written problem: Cross out the 2 in the minuend and write a 1, then write a 1 in front of the 2 to show 12 tens. Then, write a 4 below the horizontal line in the tens column to show there are 4 tens remaining.

Hundreds	Tens	Ones

6. Move to the hundreds column on the place-value mat. Refer to the written problem. It is not necessary to subtract any hundreds, and so the amount of hundreds remains the same (1). Write 1 below the horizontal line in the hundreds column. Make it clear to the student how the base-10 blocks and the written problem correspond.

Hundreds	Tens	Ones

7. Have the student read the completed problem: 234 – 88 = 146.

1	1 2	1	
	2	3	4
+		8	8
	1	4	6

Provide <u>at least </u>two more examples (e.g., 718 – 462; 803 – 250).

Guided Practice:

1. Have the student write down the multidigit subtraction problem (e.g., 739 – 215). Use graph or lined paper, if needed, to maintain place-value columns.

2. The student represents the minuend (i.e., the initial, or starting, amount: 739) on the place-value mat.

3. Beginning with the ones column, the student subtracts the ones (units), exchanging for tens (rods) if necessary. The student completes the ones column of the written problem.

4. The student repeats the procedure with the tens column, then the hundreds column.

5. The student reads the completed answer.

6. Repeat with two more examples (e.g., 932 – 87; 226 – 101).

Corrective Feedback:

Example 1

Student response: Student always subtracts the smaller number from the larger number, rather than the subtrahend (i.e., amount being subtracted) from the minuend (i.e., initial, or starting, amount). For example, for 234 – 88, student subtracts 4 from 8 in the ones column, 3 from 8 in the tens column, and answers 254.

Teacher feedback: Have the student estimate the difference before completing his or her work and then compare the estimate to the incorrect answer. Point out to the student that it is not possible to subtract an amount from a given number and end up with more than the starting amount. Have the student demonstrate the correct process before moving on.
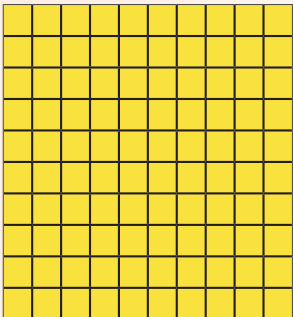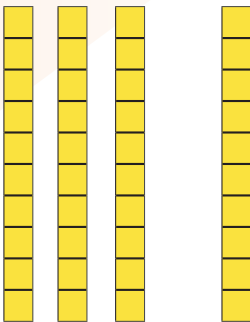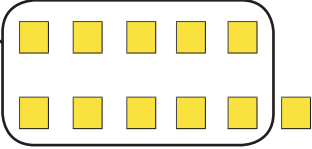
Worksheet

Place-Value Computation: Addition

Objective: Add 2- and 3-digit numbers using blocks or representations.

Directions: Write the numbers that you want to add in the place-value chart to line up their place values. Then use base-10 blocks or draw pictures to represent the blocks and solve the addition problems.

1.

$$
\begin{array}{r}
125 \\
+\ \ 16 \\
\hline
141 \\
\end{array}
$$

Hundreds	Tens	Ones
1	2	5
	1	6
1	4	1

2.

```
  142
+  27
_____
```

Hundreds	Tens	Ones

3.

```
  236
+ 126
_____
```

Hundreds	Tens	Ones

4.

$$318$$
$$+ \quad 49$$

Hundreds	Tens	Ones

5.

$$169$$
$$+ 224$$

Hundreds	Tens	Ones

6.

```
    163
+   172
```

Hundreds	Tens	Ones

7.

```
    268
+   313
```

Hundreds	Tens	Ones

8.

$$411$$
$$+ \ 196$$

Hundreds	Tens	Ones

9.

$$267$$
$$+ \ 258$$

Hundreds	Tens	Ones

10.

$$345 + 179$$

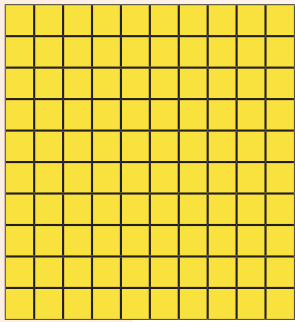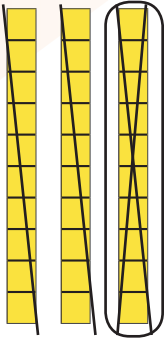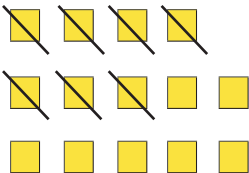| Hundreds	Tens	Ones

Worksheet

Place-Value Computation: Subtraction

Objective: Subtract 2 and 3 digit numbers using blocks or representations.

Directions: Write the numbers that you want to subtract in the place value chart to line up their place values. Then use base-10 blocks or draw pictures to represent the blocks and solve the subtraction problems. Cross out drawings or remove blocks as you subtract.

1.

```
  1 3 4
−   2 7
───────
  1 0 7
```

Hundreds	Tens	Ones
1	3	4
	2	7
1	0	7

2.

```
  1 5 5
–   4 8
```

Hundreds	Tens	Ones

3.

```
  2 7 6
–   3 7
```

Hundreds	Tens	Ones

4.

```
    161
 −   43
```

Hundreds	Tens	Ones

5.

```
    237
 − 158
```

Hundreds	Tens	Ones

6.

$$394 - 43$$

Hundreds	Tens	Ones

7.

$$312 - 168$$

Hundreds	Tens	Ones

8.

```
  1 7 1
–   9 9
```

Hundreds	Tens	Ones

9.

```
  2 2 4
–   5 6
```

Hundreds	Tens	Ones

10.

$$\begin{array}{r} 538 \\ -\ 348 \\ \hline \end{array}$$

Hundreds	Tens	Ones

3. Use Place-Value Understanding and Properties of Operations to Perform Multidigit Arithmetic

Sample Activities

    a)   Activity One: Using Base-10 Blocks for Multiplication

    b)   Activity Two: Using Base-10 Blocks for Division

Worksheets

    a)   Place-Value Computation: Multiplication

    b)   Place-Value Computation: Division

Sample Computation Concept:
Use Place-Value Understanding and Properties of Operations to Perform Multidigit Arithmetic

College and Career Ready Standards Addressed

4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations.

4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division.

Activity One: Using Base-10 Blocks for Multiplication

Purpose:

To relate concrete objects to a written method for multiplying and dividing multidigit numbers.

Principles of Intensive Intervention Illustrated:

- Provide concrete learning opportunities (including use of manipulatives).
- Provide explicit error correction, and have students repeat the correct process.
- Use precise, simple language to teach key concepts or procedures.
- Use explicit instruction and modeling with repetition to teach a concept or demonstrate steps in a process.
- Provide repeated opportunities to practice each step correctly.

Materials:

- Base-10 blocks (see Supplemental Materials)
- Place-value mat with columns for hundreds (flats), tens (rods), ones (units) (see Supplemental Materials)

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- Paper and pencil or dry-erase board
- Worksheet: Place-Value Computation: Multiplication (for extra practice)
- Example: Place-Value Computation: Multiplication (see Supplemental Materials)

Modeling:

1. Explain to the student that base-10 blocks can be used to demonstrate multiplication of multidigit numbers.

2. Write a multiplication problem (e.g., 124 × 3). Use graph paper or notebook paper turned sideways to line up place values of the factors.

$$
\begin{array}{c|c|c|c}
 & 1 & 2 & 4 \\
\hline
 & x & & 3 \\
\end{array}
$$

3. Remind student that
   - 124 × 3 can be read as "124 groups of 3."
   - Because of the commutative property of multiplication, the order of the factors can be reversed to 3 × 124.
   - 3 × 124 can be read as "3 groups of 124."

4. To show 3 groups of 124, place 3 paper plates on the table to represent the three groups.

5. Count out 124 in base-10 blocks (i.e., 1 flat, 2 rods, 4 units) for each group and place them on the paper plates; each of the 3 paper plates will have 1 flat (100), 2 rods (20), and 4 units (4).
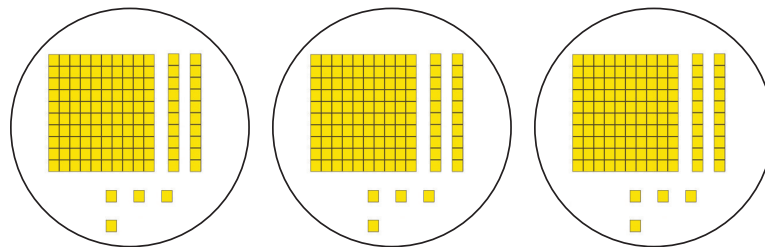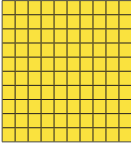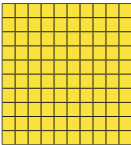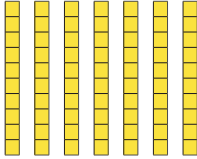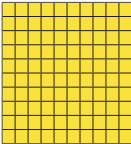
6. Use the place-value mat to represent the product. Emphasize that multiplication is repeated addition (124 × 3 = 3 × 124 = 124 + 124 + 124) and with addition, we add units in the ones column first; with multiplication, we also begin with ones. Combine the ones and place them on the mat.

7. Combining the 3 groups of 4 ones (units) yields 12 ones (units); exchange 10 ones (units) for 1 ten (rod) and move it to the tens column. On paper, write 2 in ones place below the equal sign (horizontal line) and write 1 in the tens column (this "reminder number" is a prompt to student to add the amount after multiplying in the next column), above the 2.

Hundreds	Tens	Ones
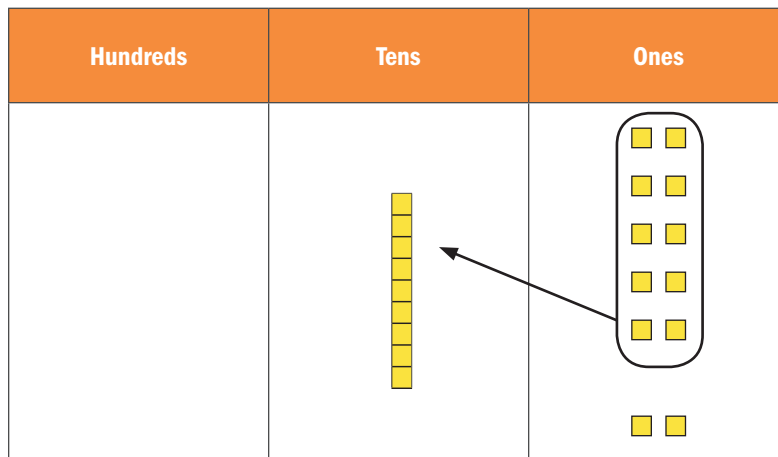
8. Next, combine all tens (rods) and place them on the mat. Write 7 in the tens column below the horizontal line to show 7 tens. Repeat with all the hundreds (flats). Write 3 below the horizontal line to show 3 hundreds. Have the student read the completed problem: 124 × 3 = 3 × 124 = 372.

	1	
1	2	4
x		3
3	7	2

Hundreds	Tens	Ones

9. Provide at least two more examples (e.g., 282 × 4; 109 × 6) using base-10 blocks to demonstrate the written procedure.

Guided Practice:

1. Have the student write down the multiplication problem (e.g., 118 × 5). Use graph or lined paper, if needed, to maintain place-value columns.

2. The student uses the commutative property to change the problem to 5 groups of 118.

3. The student places 5 paper plates on the table and uses base-10 blocks to show 118 (i.e., 1 flat, 1 rod, 8 units) on each of the 5 plates.

4. Beginning with ones (units), the student adds all the ones (units), exchanging for tens (rods) as appropriate. The student completes the ones column of the written problem.

5. The student repeats with the tens column, then the hundreds column.

6. The student reads the completed answer: 118 × 5 = 590.

7. Repeat with at least two more examples (e.g., 452 × 2; 199 × 4).

Corrective Feedback:

Example 1

Student response: Student adds the "reminder" number to the next column and then multiplies, rather than multiplying and then adding the reminder number.

Teacher feedback: Write the "reminder" number below the equal sign (horizontal line) of the written problem, to remind the student to add the number just before recording a product. Have the student demonstrate the correct process before moving on.
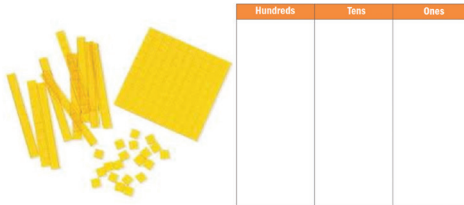
Activity Two: Using Base-10 Blocks for Division

Purpose:

To relate concrete objects to a written method for multiplying and dividing multidigit numbers.

Principles of Intensive Intervention Illustrated:

- Provide concrete learning opportunities (including use of manipulatives).
- Provide explicit error correction, and have students repeat the correct process.
- Use precise, simple language to teach key concepts or procedures.
- Use explicit instruction and modeling with repetition to teach a concept or demonstrate steps in a process.
- Provide repeated opportunities to practice each step correctly.
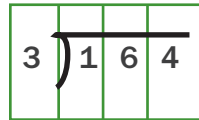
Materials:

- Base-10 blocks (see Supplemental Materials)
- Place-value mat with columns for hundreds (flats), tens (rods), ones (units) (see Supplemental Materials)

- Paper and pencil or dry-erase board
- Worksheet: Place-Value Computation: Division (for extra practice)
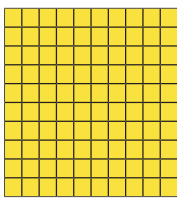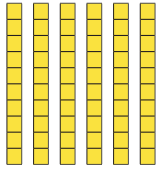- Example: Place-Value Computation: Division (see Supplemental Materials)

Modeling:

1. Explain to the student that base-10 blocks also can be used to demonstrate division of multidigit numbers.

2. Write a multidigit division problem (e.g., 164/3). Use graph paper or notebook paper turned sideways to line up place values of the dividend (amount to be divided) and divisor (amount divided by).


3. Represent the dividend (164) with base-10 blocks on the place-value mat (4 ones= 4 unites, 6 tens = 6 rods, 1 hundred = 1 flat).

Hundreds	Tens	Ones

4. Explain that the problem is to divide 164 into 3 groups. Place 3 paper plates on the table to represent the 3 groups.

5. Remind the student that in the case of division, to always start with the largest value in the dividend (i.e., the amount being divided); in this example: 100. Point to 1 flat. Emphasize that it is not possible to share 1 flat with 3 groups; the solution is to exchange 1 hundred (flat) for 10 tens (rods) and combine them with the 6 tens (rods) already on the mat. Now, share 16 tens (rods), one at a time, among the 3 groups. Each group has 5 rods, with 1 rod left over. Write a 5 above the bar over the 6 in the dividend.

Hundreds	Tens	Ones
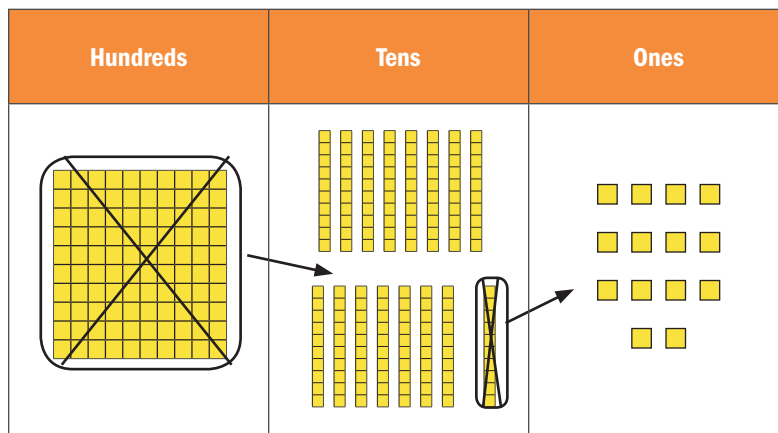
6. Emphasize that it is not possible to share 1 rod with 3 groups; the solution is to exchange 1 ten (rod) for 10 ones (units) and combine them with the 4 units already on the mat. Now, share 14 ones (units) evenly with the 3 groups. Each group has 4 units, with two units left over. Write a 4 above the bar over the 4 in the dividend.

7. Show student the 2 leftover ones (units). Have the student read the completed problem: 164 divided by 3 = 54 R 2.

8. Provide at least two more examples (e.g., 723/5; 698/4).

Guided Practice:

1. Have the student write down multidigit division problem (e.g., 607/3). Use graph or lined paper, if needed, to maintain place-value columns.

2. Student represents dividend (607) on the place-value mat; student places 3 plates on the table to represent the divisor.

3. Beginning with the hundreds, the student evenly shares among groups, exchanging for tens (rods) if necessary. The student writes the number of hundreds divided into each group on the written problem.

4. Student repeats with the tens, then the ones.

5. Student reads the completed answer.

6. Repeat with at least two more examples (e.g., 932/2 ; 226/4).

Corrective Feedback:

Example 1

Student response: Student has a difficult time remembering the multiple steps of a division algorithm while simultaneously trying to remember multiplication and division basic facts.

Teacher feedback: Provide a graphic organizer or mnemonic (e.g., **D**ad, **M**other, **S**ister, **B**rother for **Divide, Multiply, Subtract, Bring down**) to help the student remember the procedural steps. Have the student demonstrate the correct process before moving on.

Worksheet

Place-Value Computation: Multiplication

Objective: Multiply 2- and 3-digit numbers by 1-digit numbers, using manipulatives to represent the problem.

Directions:

1. Read the problem as "____ groups of ____."

2. Use paper plates for each group, then use base-10 blocks to show how many are in each group.

3. Combine all base-10 blocks in a place-value chart to find the product.

Example:

$$\begin{array}{r} 1\ 4\ 3 \\ \times\quad 3 \\ \hline 4\ 2\ 9 \end{array}$$

_3_ groups of _143_

Note: See Supplemental Materials for an example of how this can be solved using base-10 blocks.

1.

$$\begin{array}{r} 2\ 1\ 5 \\ \times\quad 2 \\ \hline \end{array}$$

____ groups of ____

2.

$$\begin{array}{r} 3\ 3\ 1 \\ \times\quad 2 \\ \hline \end{array}$$

____ groups of ____

3.

$$\begin{array}{r} 146 \\ \times\ \ \ 3 \\ \hline \end{array}$$

———— groups of ————

4.

$$\begin{array}{r} 207 \\ \times\ \ \ 3 \\ \hline \end{array}$$

———— groups of ————

5.

$$\begin{array}{r} 310 \\ \times\ \ \ 3 \\ \hline \end{array}$$

———— groups of ————

6.

$$\begin{array}{r} 453 \\ \times\ \ \ 2 \\ \hline \end{array}$$

———— groups of ————

7.

$$\begin{array}{r} 123 \\ \times\ \ \ 4 \\ \hline \end{array}$$

———— groups of ————

Worksheet: Place-Value Computation: Multiplication

8.

$$\begin{array}{r} 384 \\ \times\ \ 2 \\ \hline \end{array}$$

——— groups of ———

9.

$$\begin{array}{r} 147 \\ \times\ \ 2 \\ \hline \end{array}$$

——— groups of ———

10.

$$\begin{array}{r} 258 \\ \times\ \ 3 \\ \hline \end{array}$$

——— groups of ———

Worksheet

Place-Value Computation: Division

Objective: Divide 3-digit numbers by 1-digit numbers, using manipulatives to represent the problem.

Directions:

1. Write the problem in the grid with the dividend inside the box and the divisor outside.

2. Represent the dividend with base 10 blocks in the place-value chart.

3. Divide the dividend up into groups (divisor) on paper plates.

4. Regroup and exchange blocks when they cannot be divided evenly.

5. Write the number of hundreds, tens, and ones that are in each group in the appropriate place on the grid.

Example:

429 ÷ 3 =

		1	4	3		
	3)	4	2	9		

Note: See Supplemental Materials for an example of how this can be solved using base-10 blocks.

1. 248÷ 2 =
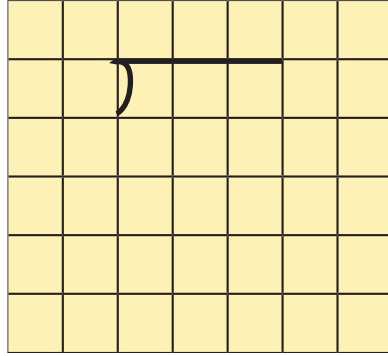
2. 356÷ 2 =
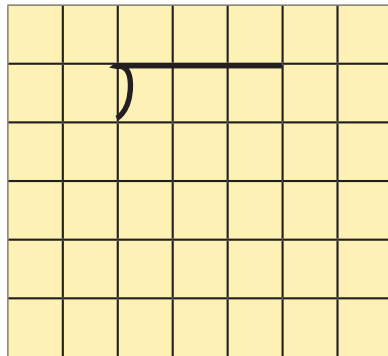
3. 192÷ 3 =

4. 410÷ 2 =
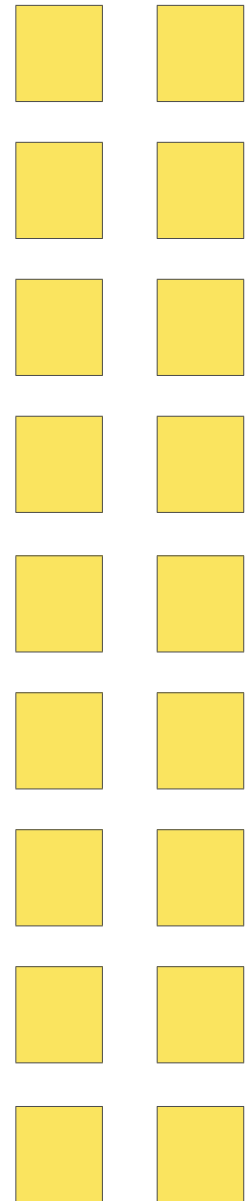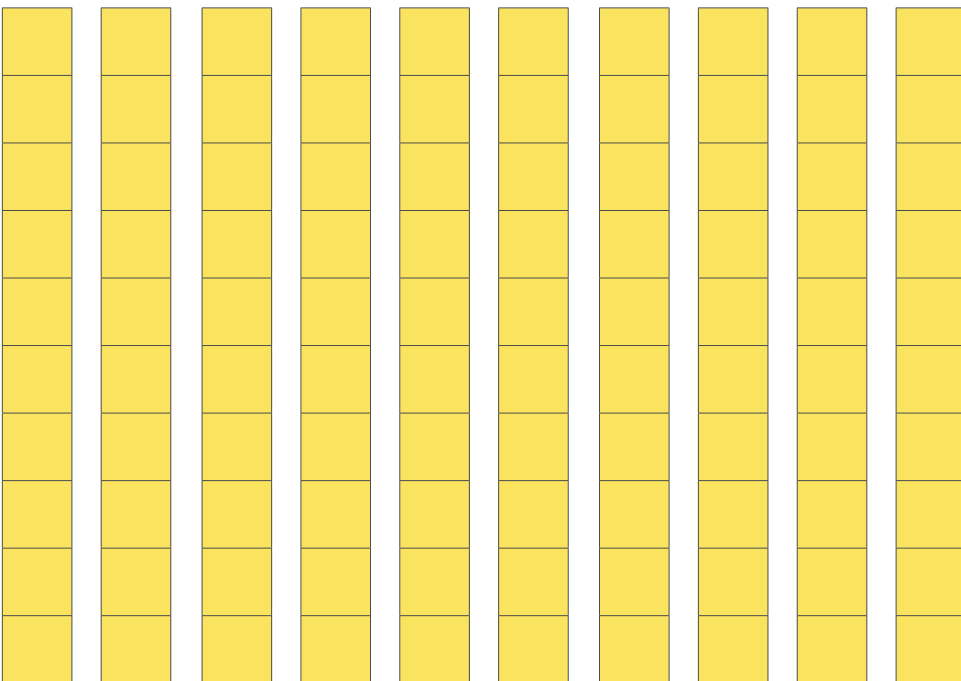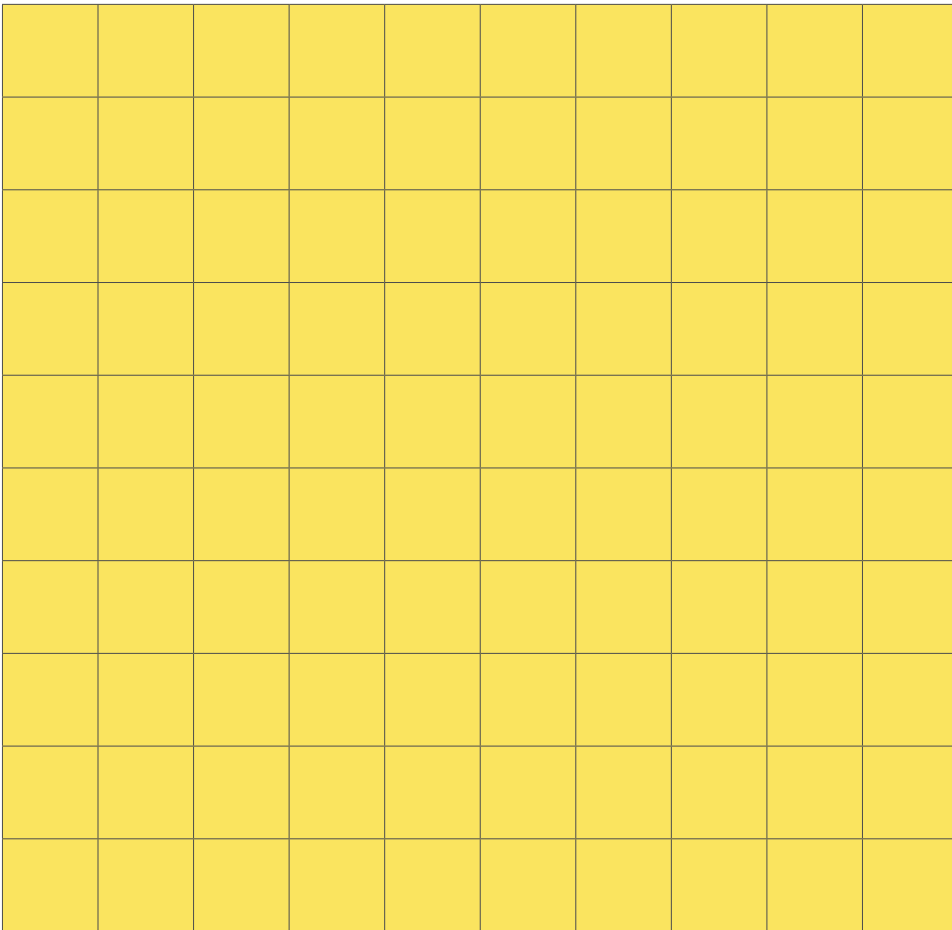
5. 516÷ 3 =

6. 411÷ 3 =

7. 428÷ 4=

8. 235÷ 5=

9. 519÷ 3=

10. 268 ÷ 4=

4. Supplemental Materials

a) Base-10 Blocks

b) Place-Value Mat: Hundreds, Tens, Ones

c) Example: Place-Value Computation: Multiplication

d) Example: Place-Value Computation: Division

Base-10 Blocks

Place-Value Mat—Hundreds, Tens, Ones

Hundreds	Tens	Ones

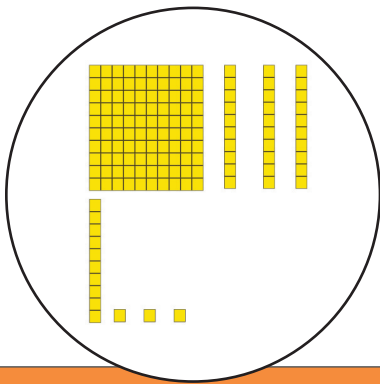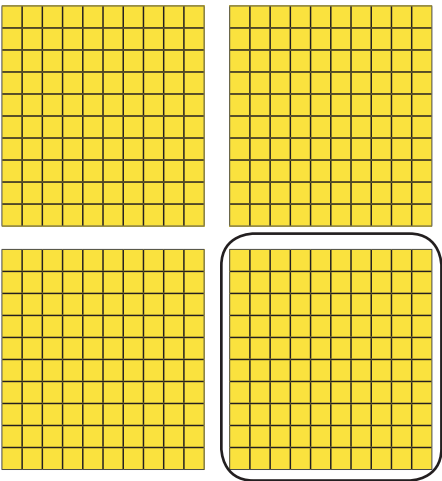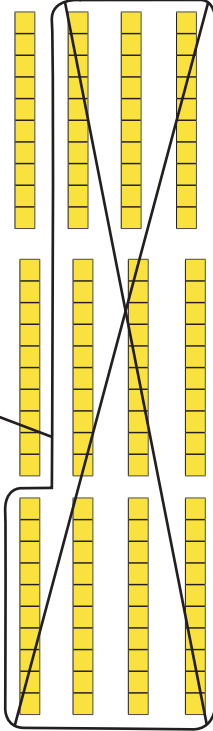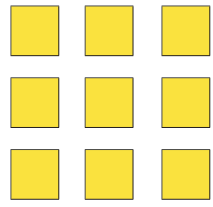# Place-Value Computation: Multiplication Example

Directions:

1. Read the problem as "_____ groups of _____."

2. Use paper plates for each group, and then use base 10 blocks to show how many are in each group.

3. Combine all base-10 blocks in a place-value chart to find the product.

Example:

$$\begin{array}{r} 1\,4\,3 \\ \times\quad 3 \\ \hline 4\,2\,9 \end{array}$$

3 groups of **143**
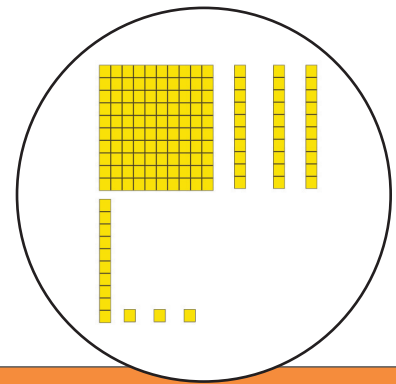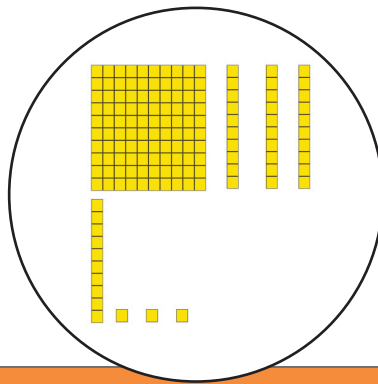
Hundreds	Tens	Ones
4	2	9

Place-Value Computation: Division Example

Directions:

1. Write the problem in the grid with the dividend inside the box and the divisor outside.

2. Represent the dividend with base-10 blocks in the place-value chart.

3. Divide the dividend up into groups (divisor) on paper plates.

4. Regroup and exchange blocks when they cannot be divided evenly.

5. Write the number of hundreds, tens, and ones that are in each group in the appropriate place on the grid.

429 ÷ 3 = 143

Hundreds	Tens	Ones