National Center on
INTENSIVE INTERVENTION
at American Institutes for Research

Intervention Taxonomy Brief: Strategic Instruction Multiplication With Regrouping Partial Products Algorithm

The goal of this brief is to provide educators with information they can use to evaluate the appropriateness of **Strategic Instruction Multiplication with Regrouping Partial Products Algorithm** for a specific student or group of students who require supplemental and intensive intervention. The brief also may be used to guide decisions about the selection or purchase of a new intervention. We envision that the brief may allow users to examine the extent to which the program aligns to the Taxonomy of Intervention Intensity, a framework used by educators to categorize interventions along key dimensions. The information included in this brief is organized along the seven dimensions of the Taxonomy of Intervention Intensity and can assist educators in answering the following questions:

- Does evidence suggest that this intervention is expected to lead to improved outcomes in the identified area of need (**strength**)?
- Will the group size, duration, structure, and frequency provide sufficient opportunities for students to respond and receive corrective feedback (**dosage**)?
- Does the intervention match the student's identified needs (**alignment**)?
- Does the intervention assist the student in generalizing target skills to general education or other tasks (**attention to transfer**)?
- Does the intervention include elements of explicit instruction (**comprehensiveness**)?
- Does the student have opportunities to develop the behavior skills necessary to be successful (**behavioral support**)?
- Can the intervention be individualized with a data-based process to meet student needs (**individualization**)?

To learn more about the Taxonomy of Intervention Intensity and find resources to support implementation, visit https://intensiveintervention.org/taxonomy-intervention-intensity.

Program Summary

The Strategic Instruction Multiplication With Regrouping Partial Products Algorithm program contains the materials needed to teach the partial products algorithm for multiplication using the Concrete-Representational-Abstract method of instruction, with an emphasis on the mathematical practices infused throughout the numbers and operations standards in most states (see Exhibit 1). State standards call for computation using strategies based on place value. One such strategy is the partial products algorithm. The materials allow for computation instruction within the context of meaningful problem situations. As students master and demonstrate an understanding of multiplication, the materials assist them in understanding its relationship to other operations. The program is for elementary students with disabilities or those who struggle

in mathematics. Students who benefitted from the program demonstrated the following deficits: lacked a sense of numbers and did not understand that multidigit numbers are not separate numerals; each digit has a unique value (47 is four tens and two ones rather than the numerals 4 and 2). Students had attempted to memorize steps to the algorithm without a sense of numbers engage in various types of error patterns. Students who participated in the program demonstrated an understanding of addition with regrouping, single-digit multiplication, and fluency in multiplication facts 1–5. The program aims to build students' sense of numbers and understanding of the multiplication operation. In addition, the program helps students understand the operation within real-life situations. Therefore, each lesson presents computation problems with words that build into word problems. As the lessons progress, students differentiate between addition, subtraction, and multiplication within word problems. This allows students to engage in mathematical practices.

Exhibit 1. Program Information

Features of program implementation	Program recommendations
Grade level(s)	Grades 4–6
Group size	1–4 students
Intervention length	18 lessons, 6 weeks
Frequency	Minimum of 3 days per week
Session duration	30 minutes
Cost	Visit https://sim.ku.edu/multiplication-regrouping-partial-products
Training	Instructor's manual has pictorial and step-by-step written directions for every lesson. Professional development can be obtained at https://docs.google.com/forms/d/e/1FAIpQLSdBnnJzm05qC-dn8hEUNwiWQAShotNj9FTMdkljK-L8wopCVw/viewform.

Evidence of Taxonomy of Intervention Intensity Dimensions

The following section presents definitions for the Taxonomy of Intervention Intensity dimensions and a summary of intervention-specific evidence for each dimension. The evidence comes from the intervention's vendor or developer. It is accurate as reported to the National Center on Intensive Intervention (NCII); it was not independently verified by NCII. Additional program evidence can be found on the NCII Tools Chart and might appear on the What Works Clearinghouse. For specific questions about the content, contact the publisher at https://kucrl.ku.edu/.

Taxonomy Dimension: Strength

Strength tells us how well the program works for students with intensive intervention needs, expressed in terms of effect sizes. Effect sizes greater than 0.25 indicate an intervention has value in improving outcomes. Effect sizes of 0.35 to 0.40 are moderate, and effect sizes of 0.50 or larger are strong (preferred).

Exhibit 2 provides the effect sizes for students in need of intensive intervention organized by domain and subdomain. These effect size data are calculated on low-achieving participants, those falling at or below the 20th percentile on pretest measures of achievement. If available, additional effect sizes for disaggregated data can be found on the NCII Tools Chart.

Exhibit 2. Strategic Instruction Multiplication With Regrouping Partial Products Algorithm Effect Sizes for Students ≤20th Percentile by Domain and Subdomain

Domain	Subdomain	Outcome measures	Effect size
Mathematics	▪ Math Computation ▪ Multiplication of Multidigit Numbers	2-minute timed curriculum-based measures	Unavailable

[a] To ensure comparability of effect size across studies, NCII uses a standard formula to calculate effect sizes across all studies and outcome measures—Hedges *g*, corrected for small-sample bias.

Taxonomy Dimension: Dosage

Dosage is the number of opportunities a student has to respond or practice and receive corrective feedback. Dosage may be impacted by the size of the instructional group, the number of minutes each session lasts, the number of student-teacher interactions built into lessons, and the number of sessions provided per week.

Assuming a group size of three students, each student in the group has an estimated 44 opportunities to respond and receive corrective feedback.

Taxonomy Dimension: Alignment

Alignment (Exhibit 3) focuses on how well the program (a) addresses the target student's full set of academic skill deficits, (b) does not address skills the target student has already mastered (extraneous skills for that student), and (c) incorporates a meaningful focus on grade appropriate curricular standards.

Exhibit 3. Alignment With Content Areas Addressed

Instructional grade level(s)	Content area addressed	Skill strands
Grade 2	Mathematics	▪ Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones. ▪ Use addition and subtraction within 100 to solve one-step word problems involving situations of adding to, taking from, putting together, and taking apart.
Grade 3	Mathematics	▪ Multiply within 100 to solve word problems in situations involving equal groups. ▪ Use place value understanding to round whole numbers to the nearest 10.
Grade 4	Mathematics	▪ Multiply two 2-digit numbers, using strategies based on place value and the properties of operations.

Taxonomy Dimension: Teaching to Promote Transfer

Attention to transfer is the extent to which an intervention is designed to help students (a) transfer the skills they learn to other formats and contexts and (b) realize connections between mastered and related skills.

For multiplying two 2-digit numbers (instructional target), two activities designed to explicitly teach for transfer are (a) multiple representations with base 10 blocks and drawings and (b) word problems with a procedural mnemonic strategy.

Activity 1. The program begins with simple word problems and a discussion of the needed operation with partially filled equations. Students solve these problems by manipulating base 10 blocks and drawings of numbers (Exhibit 4). This promotes transfer to formats in which students read word problems, think about what is happening, and translate their solution path into an equation for solving. Concrete lessons provide additional templates for the translation, which are faded in the representational lessons.

Exhibit 4. Word Problem Solving Example

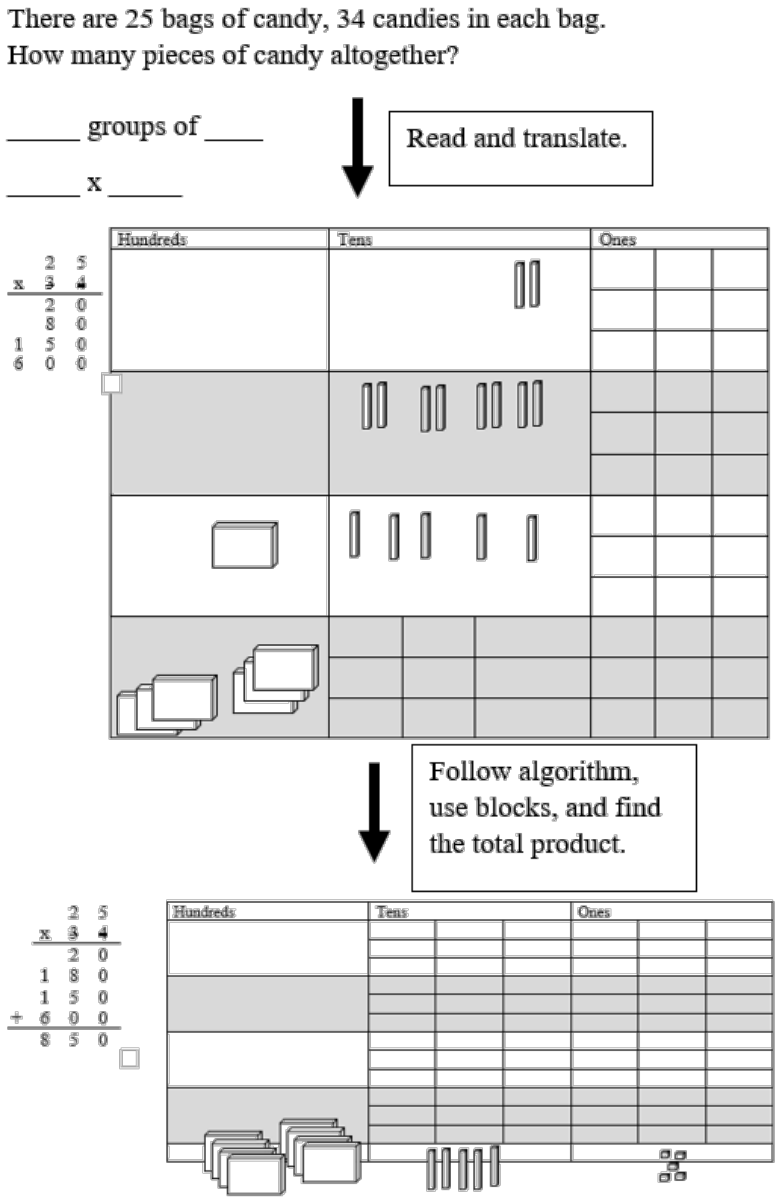
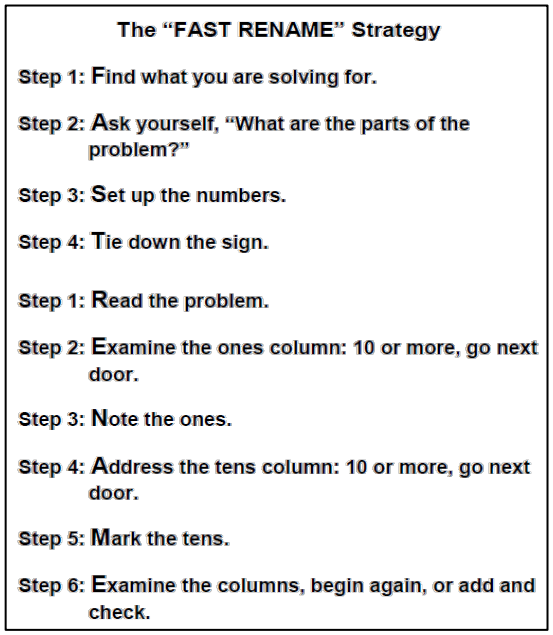


Exhibit 5. Example of Using a Strategy



Activity 2. Once students master multiplication with regrouping using objects and drawings, they solve word problems that require addition, subtraction, or multiplication using numbers only and a mnemonic strategy (Exhibit 5). This promotes transfer to other formats in which students begin with a one-step word problem. They systematically think about what is happening, using the mnemonic so that they can differentiate between operations when finding a solution.

Taxonomy Dimension: Comprehensiveness

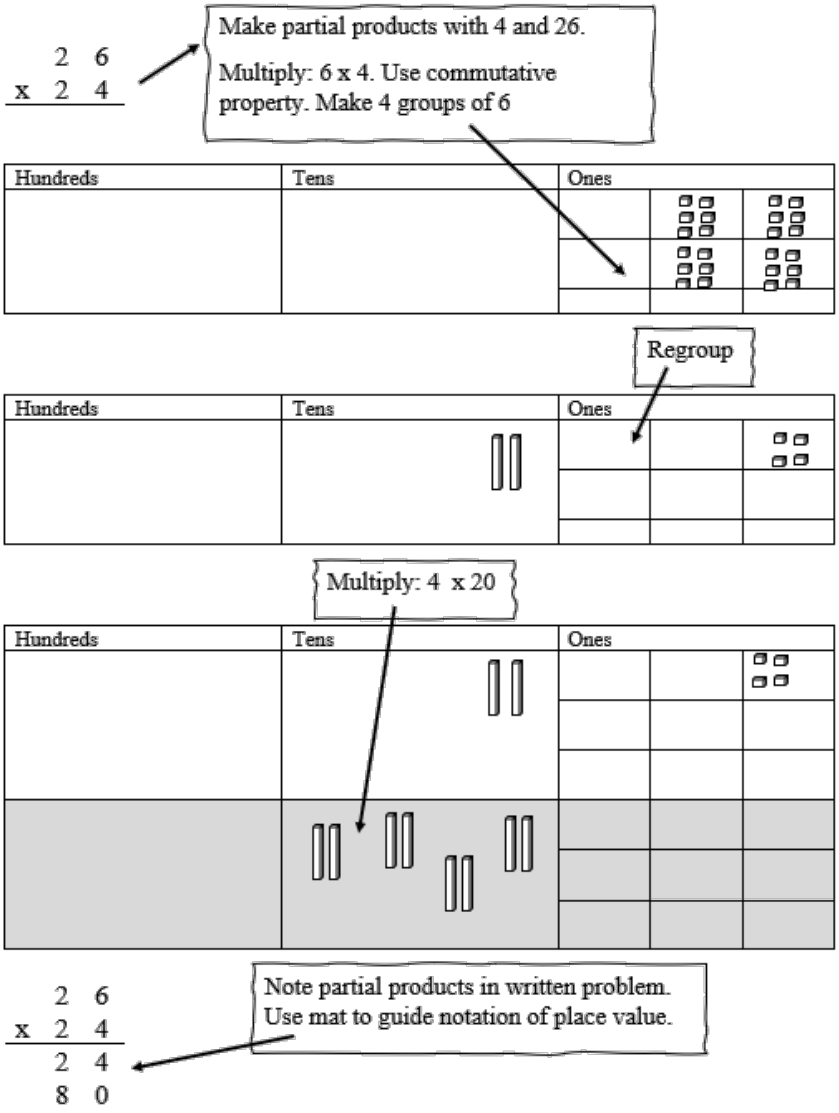
Comprehensiveness is the number of explicit instruction principles the intervention incorporates (e.g., providing explanations in simple, direct language; modeling efficient solution strategies instead of expecting students to discover strategies on their own; providing practice so students use the strategies to generate many correct responses; and incorporating systematic cumulative review). Additional information can be found within the NCII Explicit Instruction course content materials.

Dimension: Provide Explanations in Simple, Direct Language

Activity 1. Scripted lesson guides provide a model for how to present consistent and mathematically appropriate language (e.g., regrouping).

Activity 2. The use of mathematical vocabulary paired with base 10 blocks, drawings, and mats/tables helps students see and physically engage with multiplication concepts (e.g., making equal groups, the commutative property, regrouping, place value). See Exhibit 6.

Exhibit 6. Base 10 Blocks Example



Dimension: Model Efficient Solution Strategies Instead of Expecting Students to Discover Strategies on Their Own

Activity 1. The teacher physically shows each step of the computation process.

Activity 2. The teacher thinks aloud to (a) describe the steps involved in solving word problems, (b) tell about the similarities and differences between operations, (c) describe the actions or situations described in the word problems, and (d) show how to check the reasonableness of the product by rounding with mental computation. Example statements that the teacher may use during the think-alouds are as follows: "Are there two amounts and we are trying to find what is between them?" Do I have something and there is a change because something was added/lost/spent/given away?" Are we combining groups; if so, are we combining groups that are the same size or combining groups of different sizes?" See Exhibit 7.

Exhibit 7. Sample Lesson Script

Lesson Script

Now, we want to check to make sure that the product makes sense. We started with 26 x 24. We can estimate to make an easier equation and see whether our answer is close to the estimated answer. Round 24 to the nearest ten (use base ten blocks to assist if needed). Yes, 24 is close to 20. I will round 26 to the nearest ten (use base ten blocks to show if needed). I round 26 up to 30. If we multiplied 20 and 30, the product would be 600. Our product, 624; it is close to 600. Our answer seems reasonable.

Dimension: Provide Practice So That Students Use the Strategies to Generate Many Correct Responses

Activity 1. Every lesson includes modeling in which the teacher is directed to include the students (e.g., counting with the teacher, repeating information) and guided practice in which the teacher and students trade turns to solve problems. Both allow for informal assessment of student understanding and the use of prompts to ensure that students have practice with correct responses.

Activity 2. Following the modeling and guided practice, every lesson includes independent practice. The teacher support fades, allowing students to attempt computation independently. The teacher monitors students and intervenes to provide immediate feedback. Students must show that they can complete a problem with 100% accuracy without support before moving to the next lesson.

Dimension: Incorporate Systematic Cumulative Review

Activity 1. Every lesson in the program includes the same skill that becomes increasingly more complex and builds on previous lessons. Therefore, students have repeated review of and practice with the partial products algorithm. For example, lessons begin with base 10 blocks and a mat, move to drawings and a table, then to use of a procedural mnemonic strategy, and finally to application and discrimination between different operations within word problems.

Taxonomy Dimension: Behavioral Support

Behavioral support addresses the extent to which the program incorporates (a) self-regulation and executive function components and (b) behavioral principles to minimize undesired behavior. Additional information can be found within the NCII behavioral support course content.

Activity 1. The program includes student-friendly progress monitoring charts to record and track lesson progress and fluency. Each lesson includes built-in time to examine individual charts, plot current scores, and discuss where the student is compared with the goal line.

Activity 2. Each lesson begins with an advance organizer in which the teacher explains behavioral expectations for the lesson and the activities that will follow.

Activity 3. The independent practice portion of each lesson explicitly directs teacher behavior for immediate, specific feedback and assurance that the student uses the teacher's feedback (e.g., by completing task correctly according to the teacher's feedback) before moving on.

Activity 4. Each lesson has many opportunities for verbal responses and physical actions from students; the teacher recognizes each with an affirmation statement and praise (e.g. "Yes, we will make equations with 5 and the parts of 24." Or "Correct; you made four groups of thirty.").

Activity 5. At the end of each lesson, the teacher concludes with a positive statement about the students' performance and persistence throughout all lesson activities.

Additional Information About Strategic Instruction Multiplication With Regrouping Partial Products Algorithm

The aim of this program is teaching multidigit multiplication using the partial products algorithm. However, students may have gaps in their mathematical understanding (e.g., place value, meaning of multiplication). The program includes only numerals 1–5 in the multidigit numbers (e.g., 23 × 24) so that students who have not mastered all single-digit facts can participate successfully. Using base 10 blocks and drawings of numbers allows for explicit instruction and practice associated with understanding the base 10 system. The most common error pattern shown by students in pretests is treatment of multipliers as a combination of single numerals (30 × 20 is treated as 3 × 2 with the product written in the ones place. Our research has shown that older students' understanding of number composition and place value increases along with their fluency in multiplication.