## (1) Intensive Interventions 123 in Mathematics <br> 



National Center on
INTENSIVE INTERVENTION
at American Institutes for Research ■

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The purpose of this Activity Workbook is to help organize content for this Module. You will do some Activities on your own to help you engage with and think about the content. You will not be required to submit your responses for those activities. There are other activities, however, that you will submit online and apply in your classroom. The activities that you must submit before completing this Module are listed in the "Online" column below.

| Section | Assignment | To Be Completed <br> In Activity Workboolk | To Be Completed Online | To Be Completed With Coach |
| :---: | :---: | :---: | :---: | :---: |
| 은 드 | Video |  | - Watch Module 7 Introduction Video Presentation |  |
| $\begin{aligned} & \text { r } \\ & \stackrel{\rightharpoonup}{r} \\ & 0 \end{aligned}$ | Video |  | Watch Module 7 Part 1 Video Presentation |  |
|  | Activity 1 | Using the Length Model with a Student |  |  |
|  | Activity2 | - Using the Area Model with a Student |  |  |
|  | Activity 3 | - Using the Set Model with a Student |  |  |
|  | Discussion |  | Discussion Board: Favorite Fraction Tools Share Your Materials Respond to 2 Others |  |
| $N$N+O | Video |  | Watch Module 7 Part 2 Video Presentation |  |
|  | Activity 4 | - Modeling Addition and Subtraction of Fractions |  |  |
|  | Activity 5 | - Modeling Multiplication and Division of Fractions |  |  |
|  | Activity 6 | $\square$ Using CRA to Teach Decimals |  |  |
|  | Discussion |  | Discussion Board: Application to Ratio and Proportion Problems <br> Write Your Response <br> Respond to 2 Others |  |
| $\begin{aligned} & m \\ & \stackrel{ \pm}{r} \\ & \square \end{aligned}$ | Video |  | Watch Module 7 Part 3 Video Presentation |  |
|  | Activity 7 | Identify and Analyze Components of an Intervention Lesson on Fractions |  |  |
|  | Discussion |  | $\square$ Discussion Board: <br> Teaching Fractions, Decimals, or Percentages <br> $\square$ Write Your Response <br> $\square$ Respond to 2 Others |  |
|  | Video |  | Watch Module 7 Closing Video Presentation |  |
|  | Classroom Application |  |  | $\square$ Modeling Rational Number Concepts and Procedures |

- Module 7
- Part 1
- Activity \#1

Here's Abraham's work for showing the fraction $\frac{4}{5}$.


How could you use the length model to help the student understand 4/5? Draw and write your response.

- Module 7
- Part 1
- Activity \#2

Here's Bella's work for showing the fraction $\frac{4}{5}$.


How could you use the area model to help the student understand 4/5? Draw and write your response.

- Module 7
- Part 1
- Activity \#3

Here's Colleen's work for showing the fraction $\frac{4}{5}$.


How could you use the set model to help the student understand 4/5? Draw and write your response.

- Module 7
- Part 1
- Discussion


Share your favorite tools that you use when teaching about fractions. Describe why each tool is helpful and how you do modeling and practice with your students.

You could share a video or print materials.
(This space is for organizing your ideas.)
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$


1. Solve the addition problem using two different fraction models.

$$
\frac{4}{5}+\frac{1}{2}=
$$

2. Solve the subtraction problem using two different fraction models.

$$
\frac{8}{9}-\frac{2}{3}=
$$



1. Solve the multiplication problem using two different fraction models.

$$
\frac{1}{4} \times \frac{4}{6}=
$$

2. Solve the division problem using two different fraction models.

$$
\frac{7}{10} \div \frac{1}{2}=
$$



- Module 7
- Part 2
- Activity \#6

Solve this problem using any part of the concrete-representational-abstract framework.

$$
0.34 \times 3=
$$



Reflect upon the word-problem schema of ratios and proportions.
(This was covered in the module about instructional strategies, Module 4.)
How could you use some of the information in this module to help students understand ratios, unit rate, proportions, and percentages better?

Write an original post on the Discussion Board and respond to two peers.
(This space is for organizing your ideas.)

- Module 7
- Part 3
- Activity \#7

Read this intensive intervention lesson about fractions.

Identify the listed components and how they are used. (Use chart on page 22 to consolidate notes.)



Lesson Goals:


Introduce placing fractions on the Number Line

|  | Materials |  |  |
| :--- | :--- | :---: | :--- |
| $\square$ | Timer | $\square$ | R-11 |
| $\square$ | RA Checkbook | $\square$ | IC-11 |
| $\square$ | Fraction Money | $\square$ | Compare Card |
| $\square$ | Student Banks | $\square$ | Fraction Flashcards for Sprint: |
| $\square$ | WP Warm-Up Worksheet |  | Single Proper Fractions |
| $\square$ | T-11 |  |  |

## Question Guides:



Compare Card Number Line
Review Behavior Explanation: Earning Fraction Money. Review on an as-needed basis.

Start timer.


It's time for word problem warm-up!
Tutor should follow the Word Problem Visual and let students take the lead on the steps. Call on different students to help the group with underlining, circling, labeling etc. When students get in a bind, follow the types of questions/feedback below. The script below is a guide for how
 to ask questions throughout the lesson if students get stuck.

Display Word Problem Visual.

We'll solve the problem together. Let's follow the Steps on our Word Problem Card.

Listen as I read the problem.

Let's think about the story. What's happening in the story?
The candy bars are being cut. We are starting with 3 whole bars which are big and in the end we have something smaller.

Good. In this problem, something is being cut, divided, or split. Student C, what does the question say?

Student responds.
Right. This question asks "How many pieces of candy bar does Juan have now?" Juan starts with something big and at the end the pieces are smaller. Juan cuts something into smaller pieces. That's what the question asks us to find. So what kind of problem is this?

Splitting.
Right. As soon as we figure out this is a Splitting problem, we Name the Problem Type. Student A, where do we write the name of the problem type?

Above the problem.
Good. Everyone do this with me.
Tutor and students write Splitting above the word problem.
Student B, what's Step 1?


Underline what's Missing.
In a word problem, where do we find what's missing?
In the question.
Good. Student A, tell us again what the question says?
$\qquad$ ".

Student B, what's missing?
Student responds.
Good. Let's underline what's missing (demonstrate).
Student C, what's step 2?
Circle what's known.
Let's look at the word problem to see what we know. Student A, what important information is in the first sentence (point)?

Student responds.
Good. Let's circle " 3 " (demonstrate). Student B, is " 3 " the unit or the size of the pieces?
Student responds.
Good. We write U to stand for the unit.
Tutor and students label the known information as unit or size of the pieces.
Student $C$, what important information does the next sentence tell us?
$\qquad$ ."

Good. Let's circle "fourths." Student B, is "fourths" the unit or the size of the pieces?
Student responds.
Good. We write $S$ to stand for size of the pieces.
Tutor and students label the known information as unit or size of the pieces.
Student A, what's the next step?
Make a Table.
Good. Let's make the table. We draw lines like this (demonstrate). I write U here for units. I write $S$ here for the size of the pieces (demonstrate).

Tutor and students draw the two lines that make the cross and label one side with $U$ and one with $S$.

Before we fill in the table, Student A, what's missing?
How many pieces.
Good. We do not know how many pieces of "candy bar" there are. Our table will help us figure out what's missing.

Now let's fill in the table. Student A, find the " $U$ " in the problem. How many units do we write on the table?

Student responds.
Good. In our table we put " $\qquad$ " in front of U (demonstrate writing " $\qquad$ " in front of the U on the table). We write 1 here, and another one here... (continue for the number of units in problem). Everyone do that now.

Tutor and students write $a 1$ in the table for every unit. If there are 4 units, then the tutor should write four 1's

Remember, 1 unit + 1 unit... (continue for number of units in problem) makes " 3 " units. The story told us " 3 " candy bars.

Now we need to add the size of the pieces to the problem. Student B, what's the size of the pieces in our problem?

Student responds.
Remember, the "S" tells us about the size of the pieces. It does NOT tell us how many pieces. That's missing. "S" stands for the size of the missing pieces.

Write " $1 / 4$ " next to the $S$ for the size of the pieces.
Student A, how many " $1 / 4$ " pieces makes 1 unit?
Student responds.
Good. There are " 4 " pieces in each unit. Let's write $\mathrm{a} / \mathrm{b}+\mathrm{a} / \mathrm{b}+\mathrm{a} / \mathrm{b}$... (write however many pieces in 1 unit).

## SAMPLE TABLE

- The size of each piece always goes before S for these problems.

| 3 U | $1 / 4 \mathrm{~S}$ |
| :--- | :---: |
| 1 | $1 / 4+1 / 4+1 / 4+1 / 4$ |
| 1 | $1 / 4+1 / 4+1 / 4+1 / 4$ |
| 1 | $1 / 4+1 / 4+1 / 4+1 / 4$ |

- Always put a 1 under the unit column and write as many 1's as there are units.
- Write the fractions with + signs that make up the 1 unit. - The table example is for 4 units cut into fourths.

Explicit instruction

Multiple
representations

Concise language

Fluency building

Problem solving instruction

Motivation component

I finished the table. Now I Solve the Problem. Student A, remind me what's missing?
The number of pieces.
Let's count the number of pieces from our table. We count each of these fractions (point).
Tutor should point on the table to each fraction piece as the tutor and students count together. Tutor should underline each fraction as the tutor and students count together. Check off each fraction as you count it.

Now we answer the question, " $\qquad$ ?" (Tutor reads the question.)

Tutor and students should write the numerical answer and label it with the correct words from the table.

Good we answered the question. Our answer is 12 pieces of candy bar. Remember to label your answer.

## ACTIVITY 2: TRAMNiNG

What does it mean when fractions are equivalent?
They're the same amount.
Good. We've learned about fractions equivalent to 1 and $1 / 2$. How can we check if a fraction is equivalent to 1 ?

The numerator and denominator are the same.
How can we check if a fraction is equivalent to $1 / 2$ ?
The Doubling Rule.
Student B, tell me the Doubling Rule.
A fraction is equivalent to $1 / 2$ if double the numerator equals the denominator.
Good. A fraction is equivalent to $1 / 2$ if double the numerator equals the denominator.

|  |  |  |  |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
| $* \frac{3}{1}$ | $\frac{1}{11}$ | $\frac{5}{10}$ |  |
| $=\frac{1}{4}$ | $\frac{7}{6}$ | $\stackrel{¢}{4}$ |  |
| $=\frac{2}{4}$ | 4 |  |  |

TRAINING WORKSHEET SIDE A: Let's look at these fractions and decide if they're equivalent to $1 / 2$.

Explicit instruction

Multiple
representations

Concise language

Fluency building

Problem solving instruction

Motivation
component

No.
Is $2 / 3$ equivalent to $1 / 2$ ?
No.
Right. A fraction is equivalent to $1 / 2$ if doubling the numerator equals the denominator. Let's cross out $2 / 3$ to show it's NOT equivalent to $1 / 2$.

Follow these questions for the other 2 problems on the Training worksheet. Circle fractions equal to $1 / 2$ and $X$ fractions not equal to $1 / 2$.

TRAINING WORKSHEET SIDE B: Now look at these number lines. For all of these number lines, fractions equivalent to one-half are in the same place. Student A, point to $1 / 2$ on

this number line (fourths number line). Student B, point to $1 / 2$ on this number line (fifths number line).

There's not a fraction equal to $1 / 2$.
Right! You didn't let me trick you! Student C, point to $1 / 2$ on this number line (sixths number line).

## Continue through the twelfths.

Now look at the number line with just $1 / 2$ (point). On this number line, there are two equal parts between 0 and 1 (point). On this side of the number line (point), fractions are less than $1 / 2$. On that side of the number line (point), the fractions are greater than $1 / 2$. We put fractions from 0 to $1 / 2$ on this side (point). We put fractions from $1 / 2$ to 1 on that side (point). Show me which side we put fractions less than $1 / 2$.

Student points.
Great. I'll write " $L$ " above the 0 . This helps us remember fractions LESS than $1 / 2$ are on this side. Today, we'll put fractions less than $1 / 2$ on this side (point) of the $0-1$ number line.
 Show me which side of the number line we put fractions greater than $1 / 2$ ?

## Student points.

Great. We put fractions greater than $1 / 2$ on that side (point), between $1 / 2$ and 1 . I'll write " $G$ " above the 1 . This helps us remember fractions GREATER than $1 / 2$ are on this side.


TRAINING WORKSHEET SIDE C: We've been working on comparing fractions to one-half when one of those fractions equals $1 / 2$ (point to $1 / 2$ on number line). Now I'll show you how to put fractions on the number line.

This is like comparing fractions. You think about how big a fraction is and then decide where it goes on the number line. You'll think about $1 / 2$ like we did when we compared two fractions.

Look at this number line (point to top number line on T11-C). This number line goes from 0 to 1 . The number line is the unit. What fraction is already on the number line?

## $1 / 2$.

Right. We have a card to help us put fractions on the number line.

> Tutor shows Number Line card.

The first step says to find $1 / 2$. Where's $1 / 2$ ?

Students point.
3/8.
Student A, look at the Number Line card. What do we do first?
Find $1 / 2$.
Great. $1 / 2$ is marked on the number line here (point). Student B, what's the next step?
Compare to $1 / 2$ and write L or G .
To figure out where $3 / 8$ goes on the number line, we compare it to $1 / 2$. Let's write a fraction equivalent to $1 / 2$ with the same denominator. What denominator should we use?
8.

Right. We want a fraction equivalent to $1 / 2$ with 8 in the denominator because the fraction we're putting on the number line (point) has 8 in the denominator. Student C , when we have the denominator 8 , what fraction is equivalent to $1 / 2$ ?

4/8.
Good. (Write $4 / 8$ above number line where $1 / 2$ is marked.) $4 / 8$ is equivalent to $1 / 2$ so it goes in the same place on the number line. I can use the doubling rule to check it.

Now it's easy to decide. Is $3 / 8$ greater than or less than $4 / 8$ ?


Less than.
Right. $3 / 8$ is less than $1 / 2$ because $3 / 8$ is less than $4 / 8$. I write $L$ below $3 / 8$ (demonstrate). Which part of the number line is less than $1 / 2$ ?

Students point.
Good. I put an $L$ above the 0 . What part of the number line is greater than $1 / 2$ ?
Students point.
I put a G above the 1 . Everyone, point where should I put $3 / 8$ on the number line.
Students point.
Great job, everyone. $3 / 8$ is less than $4 / 8$. So $3 / 8$ is less than $1 / 2$ on the number line. $3 / 8$ goes between 0 and $1 / 2$. I make a mark on the number line where I put $3 / 8$. Then, I write 3/8 below the mark (Demonstrate on T11-C).

Now let's do the Relay!
 rule helps us decide if a fraction is equivalent to $1 / 2$ ?

The Doubling Rule.
You'll take turns. You say "equal" if the fraction is equal to $1 / 2$ and "not equal" if it is NOT equal to $1 / 2$. If you know the answer, say it fast. If not, take a few seconds to use the Doubling Rule. What happens if you say the wrong answer?

## ACTIVITY 4 FBACTION SPRINT

## Ex <br> Equal/Not Equal to 1/2: TIMED FOR 1 MINUTE

Now it's time for the Fraction Sprint. Remember, I'll hold up a fraction and you tell me if it is equivalent to $1 / 2$ or not. What

You'll tell me it's incorrect and I'll use the Doubling Rule.


Last time we did The Fraction Sprint, our group score was $\qquad$ . Work as fast as you can to beat this score. You need to beat your score each day to earn an extra bonus dollar at the end of the week!

I'm starting my timer. Begin!
Go through flashcards until the timer beeps. If a student struggles with an answer, have that student refer back the Doubling Rule.

CORRECTION PROCEDURE FOR WRONG ANSWERS (DOUBLING RULE)
What's a + a (point to numerator)?
Is the denominator $\mathbf{b}$ (point to denominator)?
Is the fraction equal or not equal to $1 / 2$ ?

Time's up! Let's count how many the group got correct in the Fraction Sprint (count as a group). Great job! Today, we got $\qquad$ correct. Let's write this on the graph (Tutor writes the number correct on the bottom of the graph).

## ACTIVITY 5: INDIVIDUAL CONTIST



Now let's begin the Individual Contest. Listen to my directions.
First, place each fraction on the 0-1 number line. Use your Number Line Card.

Then, write the greater than, less than or equal sign between the fractions. Use the Compare Card.

On the back, solve the word problem.
Any questions?
Give students time to work on individual work, encourage them to work faster if you see any student not focusing. We want kids to work approximately 3 minutes on individual work; however, we are not timing them. Try not to exceed 5 minutes. Go over the answers with the answer key.

Reveal which problems were bonus problems and have students circle the bonus problems if they got them correct. Let students know you're marking bonus problem half dollars in your checkbook.


## ACTIVITY 6: Scohisonan

Besides the bonus points you just earned for correct work, you also get bonus points for being on task. My timer went off 3 times today. Every time it beeped, you ALL were on task. So I marked a check mark for everyone on the team 3 times. You each get 3 half dollars today. You guys did a GREAT JOB! You were on task. You listened carefully, worked hard, and followed directions.

## OR

My timer went off 3 times today. You were ALL on task XX of the 3 times. So I marked a check mark for everyone in the group XX times. You each get XX half dollars. Next time, make sure you get all 3 half dollars. Remember, you all have to be on task whenever the timer rings. Always listen carefully, work hard, and follow directions.

The team earned _ half dollars today for being on task when the timer went off! So each of you get _ half dollars to put in your banks. Also, (say students names) earned half dollars for bonus problems. You also get XX half dollars.

## Hand out half dollars.

Remember, at the end of the week, you can go to the Fractions Store to buy prizes or you can save your dollars for something more expensive.

Great job everyone! See you next time.

Identify the listed components and how they are used.

| Component |  |
| :---: | :--- |
| Explicit Instruction |  |
| Multiple <br> Representations is it Used? |  |
| Concise Language |  |
| Fluency Building |  |
| Problem-Solving |  |
| Instruction |  |$\quad$| Motivation Component |
| :--- |

(For research results on this intervention, see Fuchs et al., 2013)


Make a video of yourself teaching fractions, decimals, or percentages.
Share your experience with your classmates for examples and new understanding.
Create an original post on the Discussion Board and respond to two peers.
(This space is for organizing your ideas.)
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(1) List two ways to increase your modeling of rational-number concepts (and do it!)

Model Concepts 1:

Evidence:

Model Concepts 2:
$\qquad$

Evidence:
(2) List two ways to increase your modeling of rational-number procedures (and do it!)

Model Procedures 1:

Evidence:

Model Procedures 2:

Fuchs, L. S., Schumacher, R. F., Long, J., Namkung, J., Hamlett, C. L., Cirino, P. T.,...Changas, P. (2013). Improving at-risk learner's understanding of fractions. Journal of Educational Psychology, 105, 683-700. https://doi.org/10.1037/a0032446

Siegler, R., Carpenter, T., Fennell, F., Geary, D., Lewis, J., Okamoto, Y.,...Wray, J. (2010). Developing effective fractions instruction from kindergarten through 8th grade. Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education.

