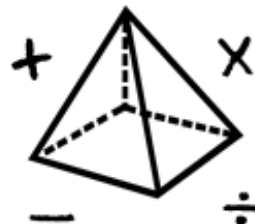




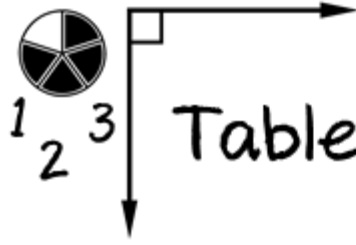
1 2 3

Intensive Interventions  
in Mathematics



# Module 6

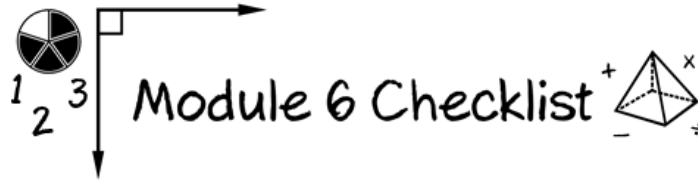
## Activity Workbook



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# Module 6 Checklist

The purpose of this Activity Workbook is to help organize content for this Module. You will do some Activities on your own to help you engage with and think about the content. You will not be required to submit your responses for those activities. There are other activities, however, that you will submit online and apply in your classroom. The activities that you must submit before completing this Module are listed in the “Online” column below.

Section	Assignment	To Be Completed In Activity Workbook	To Be Completed Online	To Be Completed With Coach
Intro	Video		<input type="checkbox"/> Watch Module 6 Introduction Video Presentation	
Part 1	Video		<input type="checkbox"/> Watch Module 6 Part 1 Video Presentation	
	Activity 1	<input type="checkbox"/> Describe Differences Between Addition and Subtraction		
	Activity 2	<input type="checkbox"/> Describe How a Student Thinks About Addition and Subtraction		
	Activity 3	<input type="checkbox"/> Describe Differences Between Multiplication and Division		
	Activity 4	<input type="checkbox"/> Describe Concepts within Multiplicative Word Problems		
Part 2	Video		<input type="checkbox"/> Watch Module 6 Part 2 Video Presentation	
	Activity 4	<input type="checkbox"/> Solve Addition and Subtraction with Two Different Algorithms		
	Activity 5	<input type="checkbox"/> Solve Multiplication and Division with Two Different Algorithms		
	Activity 6	<input type="checkbox"/> Use Two Schemas to Describe a Multiplicative Problem		
	Discussion		<input type="checkbox"/> Discussion Board: <i>Teach a Multi-Digit Operation</i> <input type="checkbox"/> Write Your Response <input type="checkbox"/> Respond to 2 Others	
Part 3	Video		<input type="checkbox"/> Watch Module 6 Part 3 Video Presentation	
	Activity 7	<input type="checkbox"/> Identify and Analyze Components of an Intervention Lesson on Computation		
	Discussion		<input type="checkbox"/> Discussion Board: <i>Non-Standard Algorithms</i> <input type="checkbox"/> Write Your Response <input type="checkbox"/> Respond to 2 Others	
Next Steps	Video		<input type="checkbox"/> Watch Module 6 Closing Video Presentation	
	Classroom Application			<input type="checkbox"/> Modeling Whole Number Concepts and Procedures



Intensive Interventions  
in Mathematics



- Module 6
- Part 1
- Activity #1

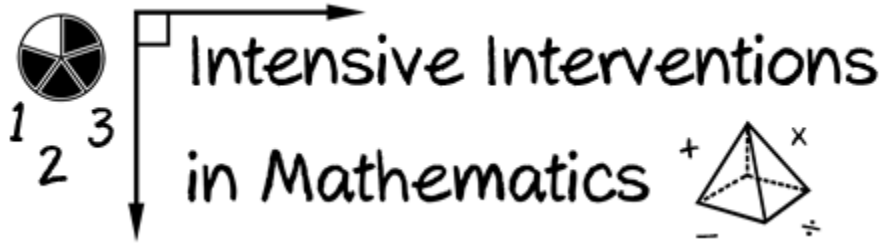



Look at problem A. Describe and draw two conceptual understandings of addition.

A.  $5 + 6 = \underline{\quad}$

Look at problem B. Describe and draw two conceptual understandings of subtraction.

B.  $8 - 3 = \underline{\quad}$



- Module 6
  - Part 1
  - Activity #2
- 

Read about how each student interprets addition and subtraction.

**1. How does Martin interpret addition?**

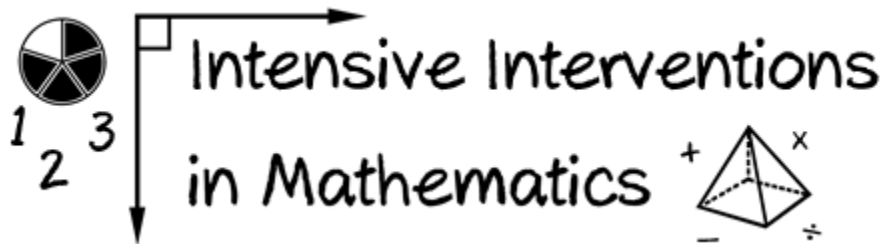
*Martin: I have 1, 2, 3, 4, 5 blue candies. I also have 1, 2, 3 green candies. If I put all the candies together, I have: 1, 2, 3, 4, 5, 6, 7, 8. I have 8 candies!*


**2. What would be another way to explain addition to Martin?**

**3. How does Lola interpret subtraction?**

*Lola: I have 1, 2, 3, 4, 5, 6, 7, 8, 9 stickers. If I put 3 stickers on my folder, I have 1, 2, 3, 4, 5, 6 stickers left.*

**4. What would be another way to explain subtraction to Lola?**




- Module 6
  - Part 1
  - Activity #3
- 

Look at problem A. Describe and draw two conceptual understandings of multiplication.


A.  $2 \times 4 = \underline{\hspace{2cm}}$

Look at problem B. Describe and draw two conceptual understandings of division.

B.  $10 \div 5 = \underline{\hspace{2cm}}$



# Intensive Interventions in Mathematics



- Module 6
- Part 1
- Activity #4



Look at these word problems.

1. What concepts of multiplication should be emphasized?
2. What concepts of division should be emphasized?

A.


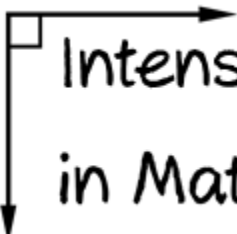
A full-grown dog weighs 7 times as much as a puppy. The puppy weighs 9 pounds. Enter the number of pounds the full-grown dog weighs. \_\_\_\_\_

B.


A teacher buys 6 bags of snack mix. Each bag contains  $2\frac{1}{2}$  cups of snack mix. The snack mix is shared evenly among 30 students. How many cups of snack mix will each student receive?

c.

Cynthia has 32 candies. She puts them into 4 bowls. Each bowl has an equal number of candies. How many candies are in each bowl?

# Intensive Interventions in Mathematics



- Module 6
- Part 2
- Activity #5



1. Solve the addition problem using two different algorithms.

$$\begin{array}{r} 256 \\ + 499 \\ \hline \end{array}$$


$$\begin{array}{r} 256 \\ + 499 \\ \hline \end{array}$$

2. Solve the subtraction problem using two different algorithms.


$$\begin{array}{r} 823 \\ - 47 \\ \hline \end{array}$$

$$\begin{array}{r} 823 \\ - 47 \\ \hline \end{array}$$





Intensive Interventions  
in Mathematics



- Module 6
- Part 2
- Activity #6



1. Solve the multiplication problem using two different algorithms.


$$\begin{array}{r} 14 \\ \times 48 \\ \hline \end{array}$$

$$\begin{array}{r} 14 \\ \times 48 \\ \hline \end{array}$$


2. Solve the division problem using two different algorithms.


$$23 \overline{)940}$$

$$23 \overline{)940}$$



# Intensive Interventions in Mathematics



- Module 6
  - Part 2
  - Activity #7
- 

Review the multiplicative schemas from the module on instructional strategies (Module 5).

How could you use two different schemas to describe this problem?

$$\begin{array}{r} 17 \\ \times 5 \\ \hline \end{array}$$

Schema 1:

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Schema 2:

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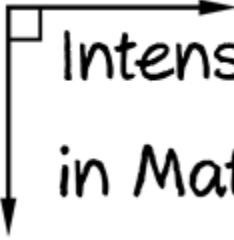

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# Intensive Interventions in Mathematics

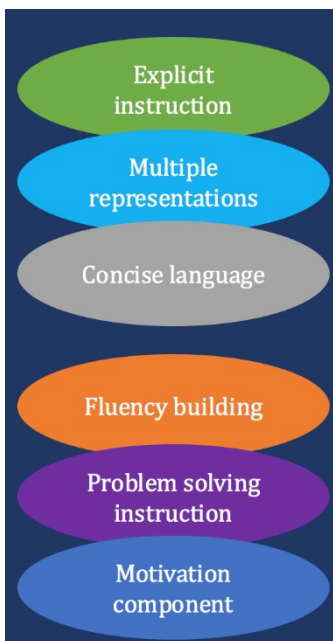


- Module 6
- Part 3
- Activity #8



Read this intensive intervention lesson about computation.

Identify the listed components and how they are used. (Use chart on page 23 to consolidate notes.)



## Lesson 17

*Today's Activities:*

1. Math Fact Flash Cards
2. Math Fact Lesson  
Counting Up Addition/Subtraction  
Combinations of 5
3. Two-Digit Computation Lesson  
Intro to Subtraction with no trading
4. Double Digit Flash Cards
5. Math Wise Review

### MATERIALS



Posters:

*Counting Up Addition Poster*      *Counting Up Subtraction Poster*



Materials:

Timers (2)	Treasure Chest
Dry Erase Marker	Pencils
Dry Erase Board	Colored Pencils
Math Fact Flash Cards	Highlighter
Double Digit Flash Cards	Combination Blocks (e.g., snap cubes)

Worksheets:

Attendance Log	Combinations of 5 Worksheet
<i>Math Wise</i> Tree	MW Review 17
Math Fact Flash Card Graph	
MW Worksheet 17	

### TUTOR LESSON

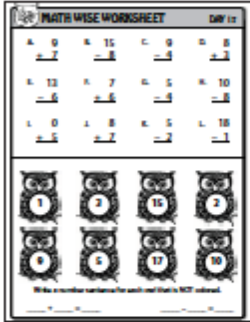
**Let's get started on *Math Wise*.**

(Set timer, without the students noticing, using the guide on the Attendance Log to determine the amount of time. Award checks as appropriate.)

### MATH FACT FLASH CARDS

## MATH FACT LESSON

Now let's work on our math fact lesson.



Pass out MW Worksheet 17.

Let's talk about counting up for addition and subtraction. (Student), how do you count up an addition problem?

Put the bigger number in your fist, count up the smaller number on your fingers, and your answer is the last number you say.

**That's right. Put the bigger number in your fist. Count up the smaller number on your fingers. Your answer is the last number you say.**

(Student), how do you count up a subtraction problem?

Put the minus number in your fist, count up to the number you started with and your answer is the number of fingers you have up.

**Very nice. Put the minus number in your fist. Count up to the number you start with. Your answer is the number of fingers you have up.**

Let's work a few problems. You may have to add or subtract. So, always look at the sign and decide what to do. Look at Problem A. (Student), do you add or subtract?

Student.

**Count up to find the answer.**

Student.

Explicit instruction

Multiple representations

Concise language

Fluency building

Problem solving instruction

Motivation component

**Write your answer.**

Student.

**Look at Problem B (point). (Student), do you add or subtract?**

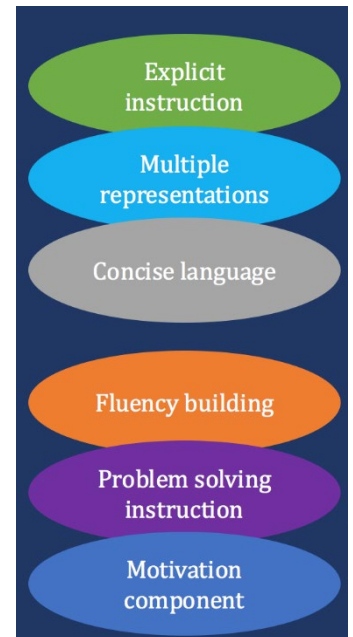
Student.

**Count up to find the answer.**

Student.

**Write your answer.**

Student.



*Continue with Problems C-F.*

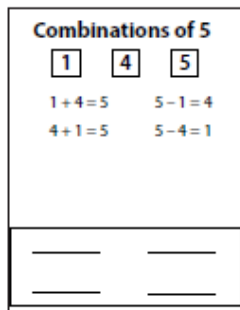
**In these problems, some are addition. Some are subtraction. So you have to watch the plus and minus signs carefully. Some problems are combinations of 5, where the biggest number is 5. For addition problems, the answer is 5. For subtraction problems, the starting number is 5. We call these problems “combinations of 5.” (Student), why do we call these problems combinations of 5?**

Because the biggest number in all of the problems is 5.

**Yes. The biggest number in all these problems is 5. That’s why we call them combinations of 5. For addition problems, 5 is the answer. For subtraction problems, 5 is the starting number.**

**Combinations of 5 go together. They can be adding or subtracting. They’re like a family. There are only a few ways to “combine” numbers together to make 5.**

**Yesterday, we worked on some combinations of 5.**



Point to Combinations of 5 Worksheet.

Today we are going to learn some more combinations. Look at this block of 5.



Show Combination Blocks.

We can take 5 blocks (put 5 blocks together of one color) and add 0 blocks. If we add 0 do we need to add any blocks at all?

No!

That's right! Adding 0 blocks means that we don't add any blocks to the 5 I have here (point). (Student), how many blocks do we have now?

5.

Yes. There are 5 blocks. So, 5 plus 0 is the same as 5. But look! We can turn these blocks around (flip block of 5 around) and see that 0 or no blocks (hold up no blocks) plus 5 (show 5 blocks of color) is also the same as 5! That's because we can switch the numbers around for adding: 5 plus 0 is the same as 5. (Student), what's another adding problem with the same numbers?

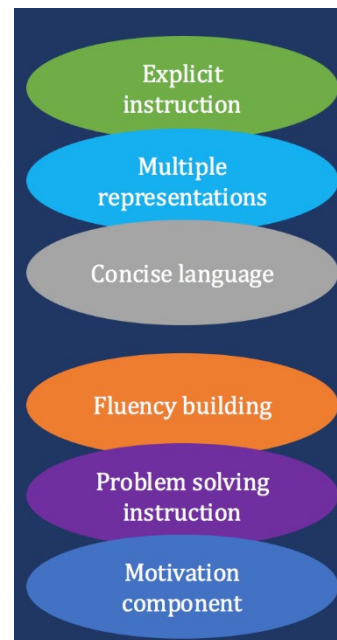
0 plus 5 equals 5.

Let's think about minus problems in the same family. We can use these blocks to show 5 (put together 5 blocks) minus 0 (take away no blocks). If we're subtracting 0, do we need to take any blocks away?

No.

That's right! 5 blocks minus 0 blocks means that we take no blocks away. The answer is the same amount of blocks we started with; 5!

(Student), how many blocks do we have now?





5.

Yes. 5 minus 0 is the same as 5. We can also start with 5 blocks (show 5 blocks) and take away 5 (take away 5 blocks). (Student), are we left with any blocks?

No!

That's right. 5 minus 5 is the same as 0.

Combinations of 5	
0	5
$0 + 5 = 5$	$5 - 0 = 5$
$5 + 0 = 5$	$5 - 5 = 0$
_____	_____
_____	_____

Display another Combinations of 5 Worksheet; 0, 5.

These four problems are in the same family. They all use the same numbers: 0 and 5. Two problems in this family are adding: 0 plus 5 is the same as 5 (point) and 5 plus 0 is the same as 5 (point). Two problems in this family are subtracting: 5 minus 0 is the same as 5 (point) and 5 minus 5 is the same as 0 (point). These four problems are a family because they use the same two numbers: 0 and 5. (Cover paper.) In each problem, the biggest number is 5. For addition problems, the answer is 5 (point). For subtraction problems, the starting number is 5 (point). On your paper, write four problems that use 0 and 5 that are in the same family.

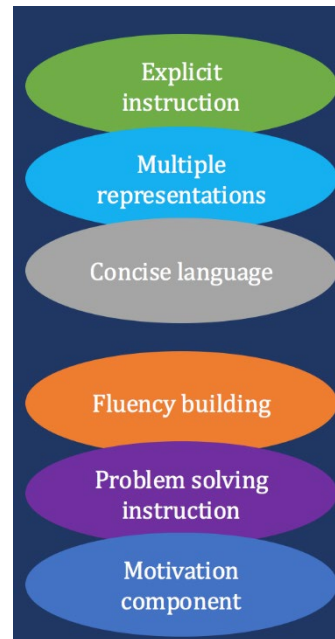
$$0 + 5 = 5; 5 + 0 = 5; 5 - 0 = 5; 5 - 5 = 0.$$

We work on combinations of 5 to become math wise. When you see an addition or subtraction problem, ask yourself, Is this a combination of 5? That can help you know the answer right off the bat.

Let's work on combinations of 5. Look at Problem G (point). (Student), what's 5 minus 4?

1.

5 minus 4 is the same as 1. Write your answer.



Student.

**Try Problem I (point). Know it or count up. (Student), what's 0 plus 5?**

5.

**5 plus 0 is the same as 5. Write your answer.**

Student.

*Continue with Problems H and J-L.*

**Let's have a combinations of 5 quiz. I'll say a math fact. You say whether it's a combination of 5 or not. Remember, in each problem, the biggest number is 5. For addition problems, the answer is 5. For subtraction problems, the starting number is 5.**

**3 minus 2? Is that a combination of 5?**

**2 plus 3? Is that a combination of 5?**

**5 minus 0?**

**5 minus 4?**

**9 minus 4?**

**2 plus 3?**

**5 plus 5?**

**10 minus 5?**

**You did a great job counting up addition and subtraction problems today. Let's practice a few problems on the owl pictures. I'll say a problem, you should know it or count up, and highlight the answer on the bottom of your worksheet. Highlight the number on the owl. Ready?**

**5 minus 4?**

**10 minus 8?**

**0 plus 5?**

**8 plus 7?**

**5 minus 2?**

**18 minus 1?**

**Now, take the two numbers not highlighted and create one subtraction number sentence and one addition number sentence.**

Student.

Explicit instruction

Multiple representations

Concise language

Fluency building

Problem solving instruction

Motivation component

## TWO-DIGIT COMPUTATION LESSON

Now, go to the back of the worksheet.

*Have the students flip worksheet over.*

**Let's work on two-digit math problems. Don't start writing until I tell you. Don't work ahead.**

**Over the past few weeks, we've worked on addition problems. For addition, we count up or compute. We've also learned how to count up subtraction problems. Today, we'll learn about subtraction problems where you compute.**

*As the tutor models two-digit subtraction, have the students work the problems on his/her own worksheet.*

**(Student), how do you know if a problem is addition?**

There's a plus sign.

**Very good. A plus sign tells you to add. (Student), how do you know if a problem is subtraction?**

There's a minus sign.

**Right! A minus sign tells you to subtract.**

**Last week, we learned how to count up subtraction problems. Now let's look at how to *compute* a subtraction problem. (Student), how do you know if you need to compute?**

If the problem has two two-digit numbers.

**That's right. You compute problems for two two-digit numbers. When you compute, you use two steps.**

**Look at Problem A on your worksheet. (Point to A.) 54 minus 32. (Student), do you add or subtract?**

Subtract.

Explicit instruction

Multiple representations

Concise language

Fluency building

Problem solving instruction

Motivation component

**Yes. How do you know to subtract?**

There is a minus sign.

**Yes. The minus sign tells you to subtract. (Student), do you count up or compute?**

Compute.

**Very good. How do you know you compute?**

There are two two-digit numbers.

**You compute because we have two two-digit numbers. 54 is a two-digit number. 32 is a two-digit number. So, we need to compute.**

**Just like with addition, when you compute, you use two steps. You subtract the ones column first. Then, in another step, you subtract the tens column. When we compute, what do we draw?**

A line.

**That's right. We draw a line between the ones and tens columns.**

*Draw a line. Monitor that the students draw a line on their own worksheets.*

**So, our ones are lined up in this ones column** (point to ones column). **The tens are lined up in this tens column** (point to tens column).

**(Student), do we start in the ones column or in the tens column?**

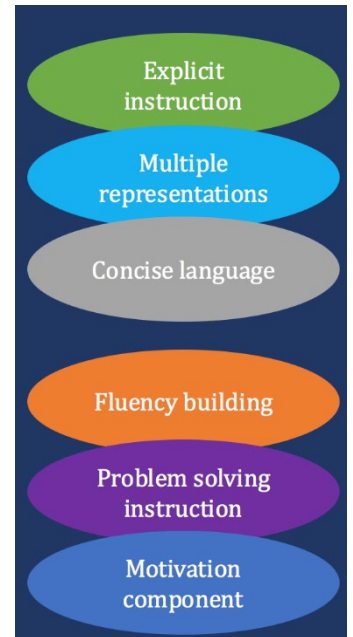
The ones column.

**Is that the same thing we do with addition problems?**

Yes.

**That's right. Whether the problem is addition or subtraction, *always* start in the ones column. Then move to the tens column.**

**(Student), what numbers are in the ones column?**



4 and 2.

**(Student), what's the number you start with?**

4.

**Yes. 4 is the number you start with. It's the number on top. (Student), what's the minus number?**

2.

**2 is the minus number. The minus number is next to the minus sign (point). So we do 4 minus 2. (Student), what's 4 minus 2? Know it or count up.**

2.

**Yes. 4 minus 2 is the same as 2. So, we write a 2 in the ones column below the equal line.**

*Write 2. Monitor that the students write 2 in the ones column.*

**We subtracted the ones column. (Student), where do we move next?**

To the tens column.

**Yes. We move over to the tens column. (Student), what numbers are in the tens column?**

5 and 3.

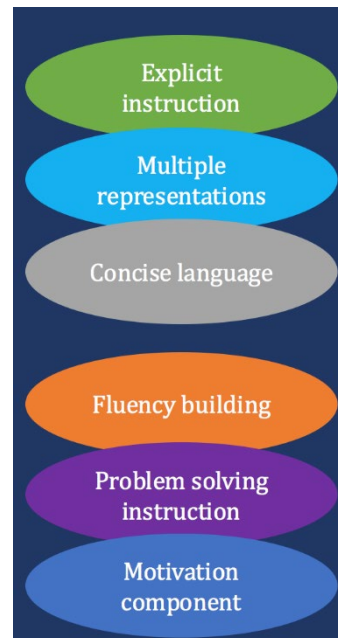
**(Student), what's the number you start with?**

5.

**(Student), what's the minus number?**

3.

**So, do 5 minus 3. (Student), what's 5 minus 3? Know it or count up.**





2.

**Nice work. 5 minus 3 is the same as 2. So, we write 2 in the tens column below the equal line.**

*Write 2. Monitor that the students write 2 in the tens column.*

**To know the answer to 54 minus 32, look at the ones and tens of the answer. In the answer, we have 2 tens and 2 ones (point). 2 tens and 2 ones is the same as 22.**

**What's 54 minus 32?**

22.

**Yes. 54 minus 32 is the same as 22.**

**Let's try another problem together. Look at Problem B on your worksheet (point). 96 minus 54. (Student), what do we ask ourselves first?**

Do you add or subtract.

**Good job. So, do you add or subtract?**

Subtract.

**How do you know to subtract?**

There is a minus sign.

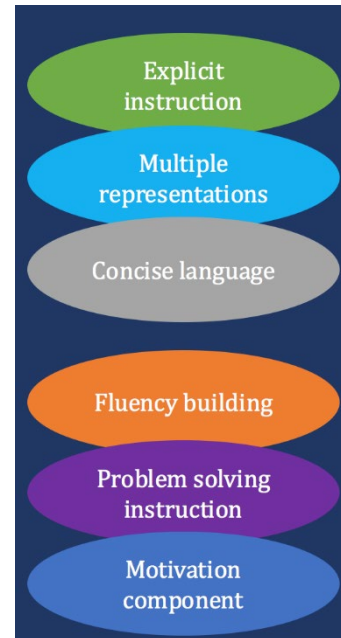
**Yes, the minus sign tells us to subtract. (Student), do you count up or compute?**

Compute.

**That's right. How do you know you compute?**

Because there are two two-digit numbers.

**Good. Whenever you have two two-digit numbers, compute. You draw a line between the tens and ones column.**



*Draw a line. Monitor that the students draw a line between columns.*

**Where do you always start: ones column or tens column?**

Ones column.

**(Student), what numbers do you subtract in the ones column?**

6 minus 4.

**That's right. You always start with the top number. So it's 6 minus 4. (Student), what's 6 minus 4? Know it or count up.**

2.

**That's right. 6 minus 4 is the same as 2.**

4.

**Bravo! So, 6 minus 4 is the same as 2. (Student), where should we write 2?**

In the ones column under the equal line.

**Good. Write 2 under the ones column.**

*Write 2 in ones column. Monitor that the students write 2 in the ones column.*

**We just subtracted the ones column. Where do we move next?**

To the tens column.

**(Student), what numbers do you subtract in the tens column?**

9 minus 5.

**(Student), what's 9 minus 5? Know it or count up.**

4.

**Great! So, 9 minus 5 is the same as 4. Where should we write 4?**

Explicit instruction

Multiple representations

Concise language

Fluency building

Problem solving instruction

Motivation component

In the tens column under the equal line.

**Good. Write 4 in the tens column under the equal line.**

*Write 4. Monitor that the students write 4 in the tens column.*

**What's 96 minus 54?**

42.

**Very good! 96 minus 54 is the same as 42.**

*Continue with Problems C-F.*

**Nice work! Computing subtraction problems is a lot like computing addition problems. I think you'll do a nice job with these problems!**

### **DOUBLE DIGIT FLASH CARDS**

*Follow Activity Guide: Double Digit Flash Cards*

### **MATH WISE REVIEW**

*Follow Activity Guide: Math Wise Review*

**Identify the listed**

**components and how they are used.**

<b>Component</b>	<b>How is it Used?</b>
<b>Explicit Instruction</b>	
<b>Multiple Representations</b>	
<b>Concise Language</b>	
<b>Fluency Building</b>	
<b>Problem-Solving Instruction</b>	
<b>Motivation Component</b>	







**(1) List two ways to increase your modeling of whole-number concepts (and do it!)**

**Model Concepts 1:**

---

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Evidence:

---

**Model Concepts 2:**

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Evidence:

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**(2) List two ways to increase your modeling of whole-number procedures (and do it!)**

**Model Procedures 1:**

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Evidence:

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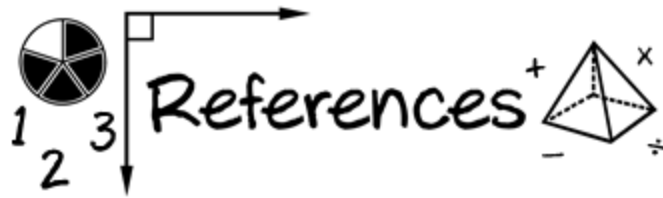
**Model Procedures 2:**

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Evidence:

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- Fuchs, L. S., Powell, S. R., Cirino, P. T., Schumacher, R. F., Marrin, S., Hamlett, C. L., ... Changas, P. C. (2014). Does calculation or word-problem instruction provide a stronger route to pre-algebraic knowledge? *Journal of Educational Psychology, 106*, 990–1006. <https://doi.org/10.1037/a0036793>
- Powell, S. R., & Fuchs, L. S. (2012). Early numerical competencies and students with mathematics difficulty. *Focus on Exceptional Children, 44*(5), 1–16.