Making Fractions Make Sense

Considerations for Secondary and Intensive Intervention

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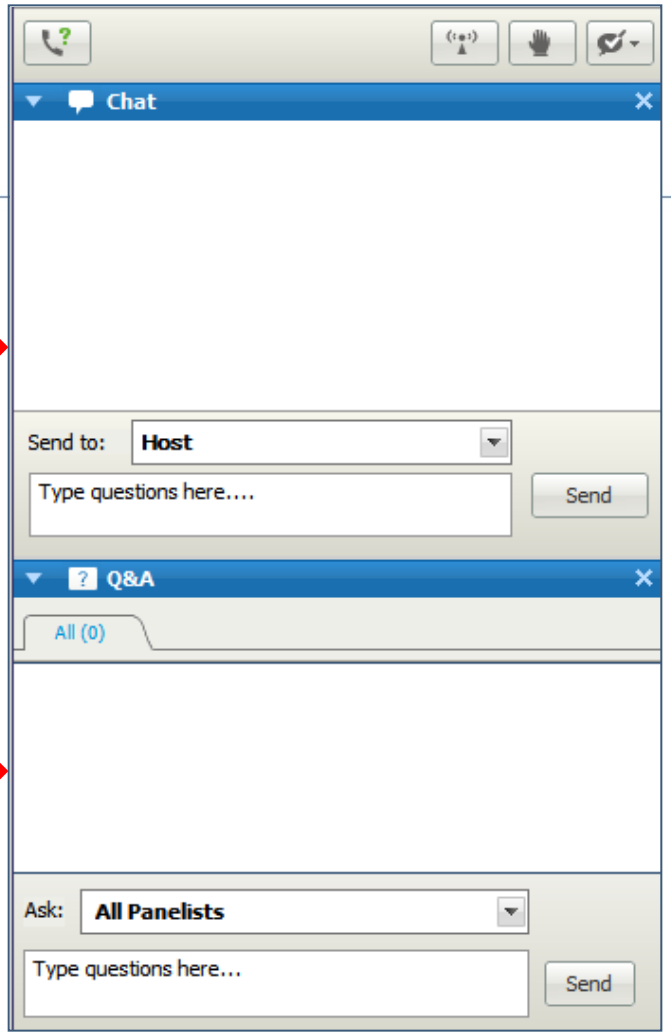
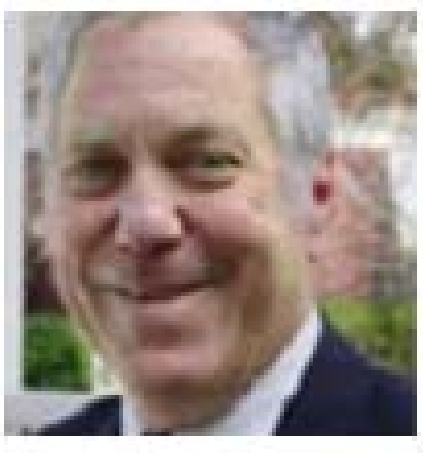


Presenters



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The Case for Emphasizing Fractions in Intervention

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Represents the Research Of

- Center for Improving the Learning of Fractions https://sites.google.com/a/udel.edu/fractions/home

- Institute of Education Sciences Practice Guide on Fractions http://ies.ed.gov/ncee/wwc/practiceguide.aspx?sid=15

The Case for Emphasizing Fractions in Intervention

Case for Emphasizing Fractions

1. Fractions knowledge (understanding and procedural but especially understanding of the ideas) is critical for success in algebra (National Mathematics Panel, 2009) mathematically.
2. Reason is that fractions opens up a level of abstraction necessary for future mathematics.
3. Can only be done by demonstrating understanding.

Why Is This Important?

It appears that if you can't do these types of problems well, algebra success is unlikely.

Sources: Siegler, Duncan et al. (2012). *Using longitudinal data from U.S. & UK*

Nationally representative sample of algebra teachers (NMAP, 2008)

National Mathematics Advisory Panel. (2008). *Mathematical analysis*

Poll Item

In which of the following are the three fractions arranged from least to greatest?

A. $\frac{5}{9}, \frac{1}{2}, \frac{2}{7}$

B. $\frac{5}{9}, \frac{2}{7}, \frac{1}{2}$

C. $\frac{2}{7}, \frac{1}{2}, \frac{5}{9}$

D. $\frac{1}{2}, \frac{2}{7}, \frac{5}{9}$

E. $\frac{1}{2}, \frac{5}{9}, \frac{2}{7}$

That Was a NAEP Item

- Eighth graders in 2007 only correctly solved this problem 49 percent of the time!
- Demonstrates critical importance of magnitude of fractions and how complex it is.

U.S. Children and Adults Have Particularly Poor Fractions Knowledge

- "Is 13/15 x 12/17 > 13/15?"
  - Sixth and eighth graders: 30 percent correct
  - Preservice teachers: 30 percent correct (real danger sign)
  - Carnegie Mellon mathematics/science students: 95 percent correct
  - Only 50 percent of eighth graders correctly ordered 2/7, 1/12, and 5/9 (NAEP, 2007)
  - Only 29 percent of 11th graders correctly translated 0.029 as 29/1000 on NAEP

(Siegler & Lortie-Forgues, in preparation)

Relations Between Fraction Magnitude Representations and Mathematics Achievement Scores: Eighth Graders

Measure of Magnitude	Mathematics Achievement
Number line 0–1 PAE	**–0.63****
Number line 0–5 PAE	**–0.86****
Magnitude comparison accuracy	**0 .62****

***p* < .01

Why Are Fractions So Hard for So Many?

- Fractions usually look like two numbers but are really one number.
  - Some students, for example, may not think of 7/3 as one number but as a "bunch of numbers."
  - One number can be represented in an infinite number of ways (e.g., 3/4, 36/48, 75/100).

Why Are Fractions So Hard for So Many?

- The same number can look quite different to the naked eye (e.g., 14/21 and 2/3), yet they are precisely the same number with precisely the same magnitude.
- Often when numerals get bigger, the value of the fraction gets smaller (e.g., 1/3 is bigger than 1/8) BUT not ALWAYS.
  - 2/5 smaller than 8/11.
  - 1/4 is the same as 5/20.

Incorrect Whole Number Strategies

- Students used two main whole number strategies to solve problems with like denominators.

Strategy	Example
Independent whole numbers	$\frac{3}{6} + \frac{1}{6} = \frac{4}{12}$ $\frac{3}{4} - \frac{1}{4} = \frac{2}{0}$
"Add all"	$\frac{3}{6} + \frac{1}{6} = 16$

(Jordan et al, 2013)

Why Is Early Fraction Knowledge Uniquely Predictive of Later, More Advanced, Mathematic Achievement?

- Fractions, including ratios and proportions, are heavily used in high school mathematics—algebra, geometry, trigonometry, etc.
- Fractions involve a level of abstraction not encountered in work with whole numbers.

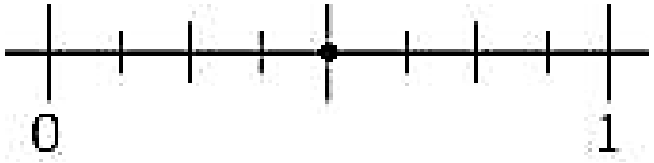
(Siegler et al., 2012)

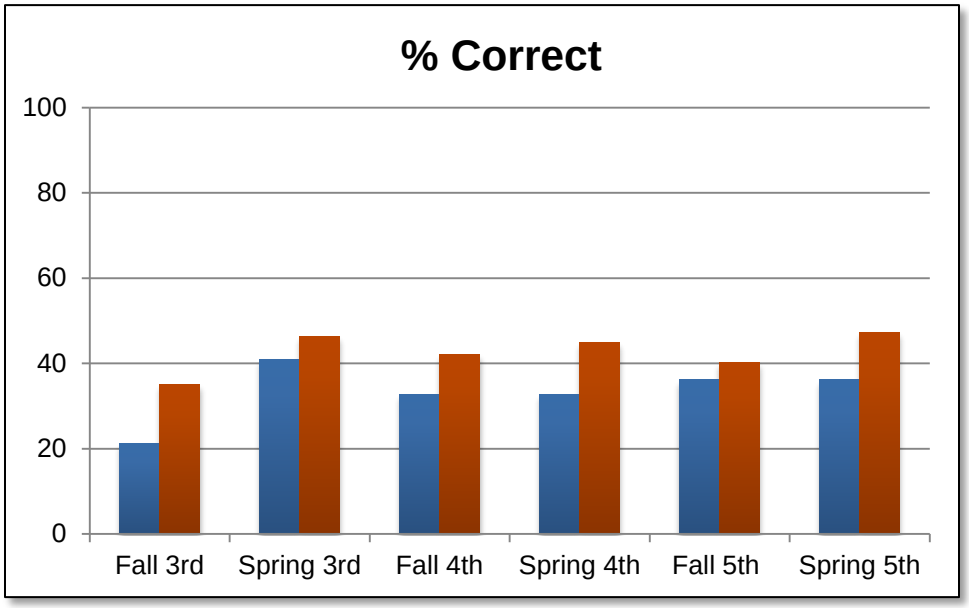
Grappling With Infinity

- With whole numbers, students learn to find “how many numbers are there between, say 8 and 10 OR 7 and 11.”
- An infinite number of numbers exist between two fractions (e.g., 1/5 and 1/2).

Fractions on the Number Line

On the portion of the number line below, a dot shows where 1/2 is. Use another dot to show where **3/4** is.

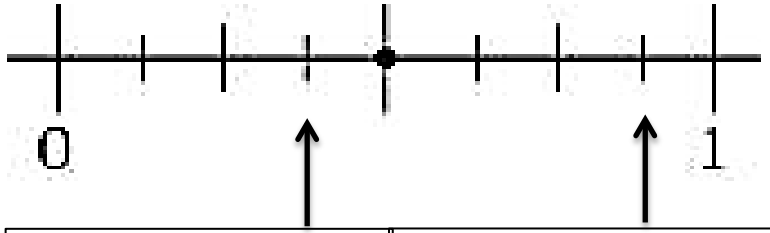




> 36th percentile in mathematics achievement

≤ 35th percentile in mathematics achievement

Common Errors



~10%-15% of kids in both groups in fourth and fifth grade	~20%-30% of kids in both groups in fourth and fifth grade

Even by the end of fifth grade, less than half of students can correctly identify fractions on the number line.

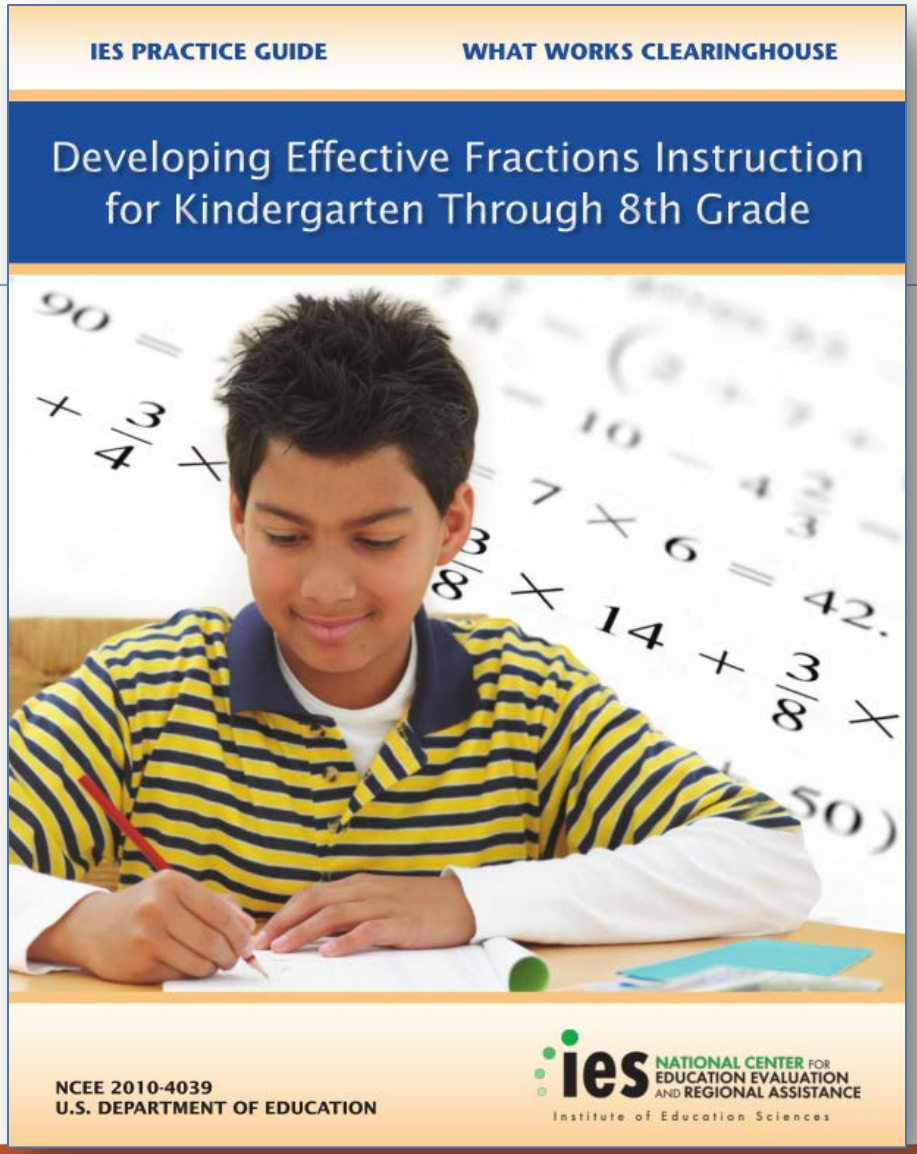
This points toward the need to further develop the understanding of a fraction as a location on a number line.

Sense of Betrayal

- Students learn fractions as part of a whole in Grade 3.
- Example:
  - Half of the class went to museum the first day. There are 18 students in the class. How many went?
  - Put 9/4 on a number line.

Mathematics Fractions Practice Guide

http://ies.ed.gov/ncee/wwc/practiceguide.aspx?sid=15



Levels of Evidence

Recommendation	Level of Scientific Evidence
Build on students' informal understanding of sharing and proportionality to develop initial fraction concepts.	Minimal
Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward.	Moderate
Help students understand why procedures for computations with fractions make sense.	Moderate
Develop students' conceptual understanding of strategies for solving ratio, rate, and proportion problems before exposing them to cross-multiplication as a procedure to use to solve such problems.	Minimal
Professional development programs should place a high priority on improving teachers' understanding of fractions and of how to teach them.	Minimal

What Prerequisite Skills Do Students Need Before They Encounter Fractions?

It is important for students to have fluent fact mastery so that they can execute fraction procedures correctly.

Second and Third Grade CCCS

Fluently add and subtract within 20 by the end of second grade.

Fluently multiply within 100 by the end of third grade.

So What Does This Mean For Struggling Students?

- Recommendations from practice guides remain important across secondary (Tier 2) and intensive intervention.
- Intensive intervention content for Grades 5 and up must include this material.
- Excellent if it is linked to grade-level content (e.g., fractions computation in Grade 5, proportions in Grade 6, and simple linear equations in Grade 7).

Evidence-Based Fraction Intervention at Fourth Grade: Tier 2

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Big Ideas

- Our goal: Build conceptual understanding of fractions as numbers; focus on two interpretations.
- Primary focus: Measurement understanding
  - Number lines
  - Magnitude; ability to reason about size
  - Infinite equivalencies
  - Focus of instruction in Asian countries
- Secondary focus: Part-whole understanding
  - Shaded regions
  - Focus of instruction in United States

Fraction Skills Addressed

- Understanding fractions as numbers
  - Naming fractions from regions
  - Fraction equivalencies to ½ and 1 whole (quick retrieval)
  - Fraction equivalencies with multiplication
  - Identify proper, improper, and mixed numbers (less than 2) + converting

Fraction Skills Addressed

- Magnitude activities
  - Comparing fractions (2) with <, >, =
  - Ordering fractions (3)
  - Number line 0–1 (place two fractions on number line)
  - Number line 0–2 (place one fraction on the number line)

Fraction Skills Addressed

- Fraction calculations
  - Simple addition and subtraction
    - Same denominators
    - Different denominators (only one fraction to change)

How Do You Think About Fractions?

Question 1:

- Compare: 4/6 and 5/12

How Do You Think About Fractions?

Question 1:

- Compare: 4/6 and 5/12
  A. Did you find a common denominator?
  B. Did you think about magnitude of each fraction versus a benchmark fraction?

How Do You Think About Fractions?

Question 2:

- Where would you decide to place 7/12 on a 0–1 number line?

How Do You Think About Fractions?

Question 2:

- Where would you decide to place 7/12 on a 0–1 number line?
  A. Did you mentally divide the number line into 12 equal parts?
  B. Did you think about ½ (6/12) to approximate where 7/12 goes?

Instructional Design

1. Introduce concept with manipulatives/visuals
   - Fraction circles, fraction tiles, number lines
2. When relevant, provide context
   - Equal sharing example for unit-fraction understanding
3. Provide procedures for solving each task
   - Decrease demand on working memory, gradually fade prompt cards
4. Include fluency practice for foundational skills
5. Independent practice to demonstrate learning

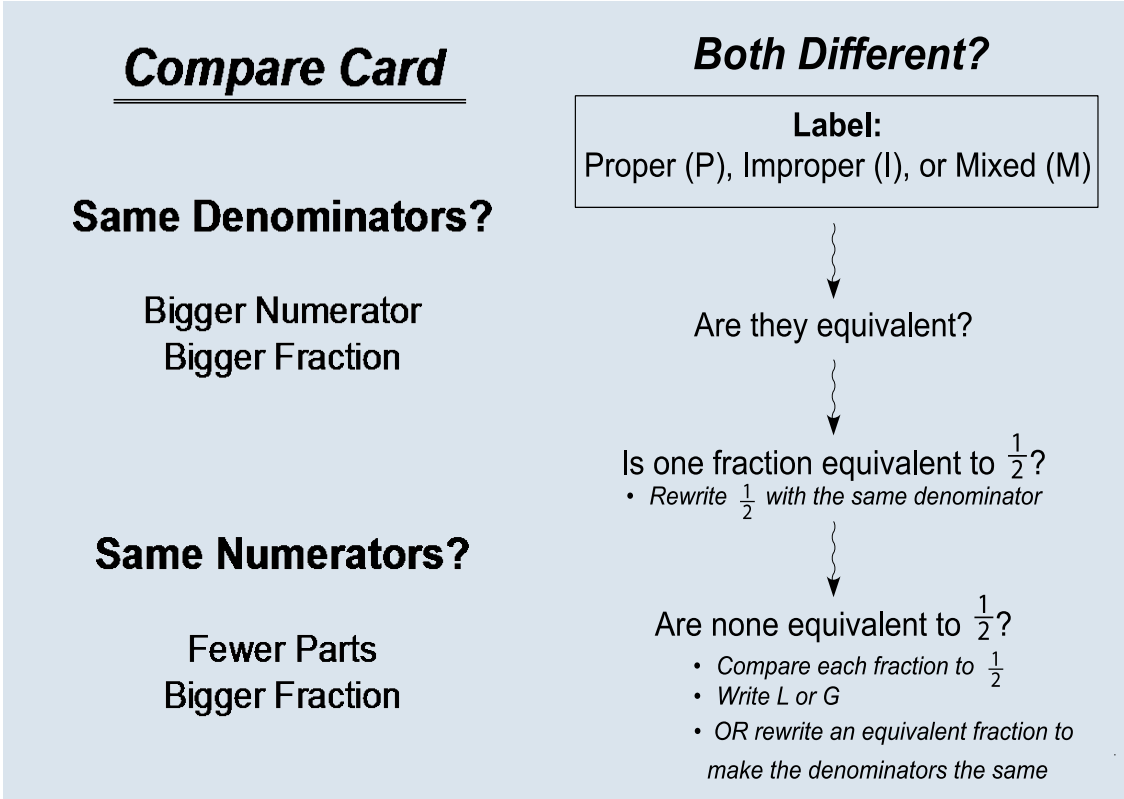
Fraction Skill: Equivalency (½ and 1 Whole)

- Introduce
  - Fraction tiles
  - Fraction circles
  - Number lines with varying denominators
  - Football analogy (quarters and half time)

- Build Understanding
  - Show it with multiplication
  - Doubling rule (double the numerator equals denominator)
- Goal: Quick Retrieval
  - Able to use as benchmark for evaluating magnitude in other activities: Transitive property

Fraction Skill: Comparing

- Introduce with fraction circles and tiles
- Build understanding
  - "equal sharing" with context of same numerators
  - "number of pieces" with same denominators
- Use ½ as a benchmark fraction for determining magnitude relationship
- Transitive property



Fraction Skill: Ordering

Ordering

$\frac{a}{d}$ $\frac{b}{e}$ $\frac{c}{f}$

Label:
Proper (P), Improper (I), or Mixed (M)
↓
Change I to M

Compare:

Same Denominators?
Bigger Numerator
Bigger Fraction

Same Numerators?
Fewer Parts
Bigger Fraction

All Different?

1. Compare to $\frac{1}{2}$ ***and***
   Write L, G, or =

2. LL or GG?
   Compare and write < or >

Order fractions from smallest to largest.

A. $\frac{3}{4}$ $\frac{1}{2}$ $\frac{2}{6}$ ____ ____ ____

B. $\frac{1}{4}$ $\frac{1}{10}$ $\frac{1}{2}$ ____ ____ ____

C. $\frac{1}{2}$ $\frac{8}{12}$ $\frac{3}{8}$ ____ ____ ____

Fraction Skill: Number Line

Number Lines

Which number line?

Find $\frac{1}{2}$

↓

Compare to $\frac{1}{2}$ and write L or G

↓

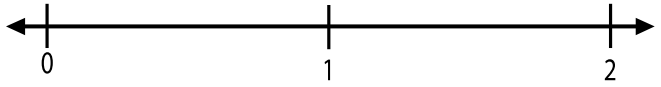
LL or GG?
Compare and write < or >

Label:
Proper(P), Improper(I), or Mixed(M)
↓
Change I to M

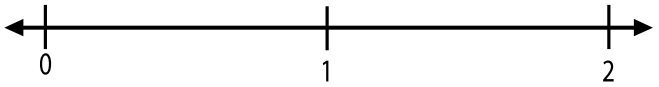
↓

If Proper:
Look at 0-1
Compare to $\frac{1}{2}$ and write L or G

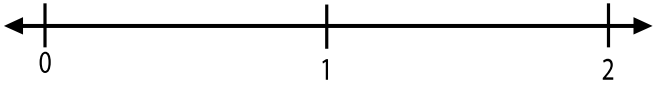
If Mixed:
Look at 1-2
Compare to $1\frac{1}{2}$



$\frac{7}{10}$



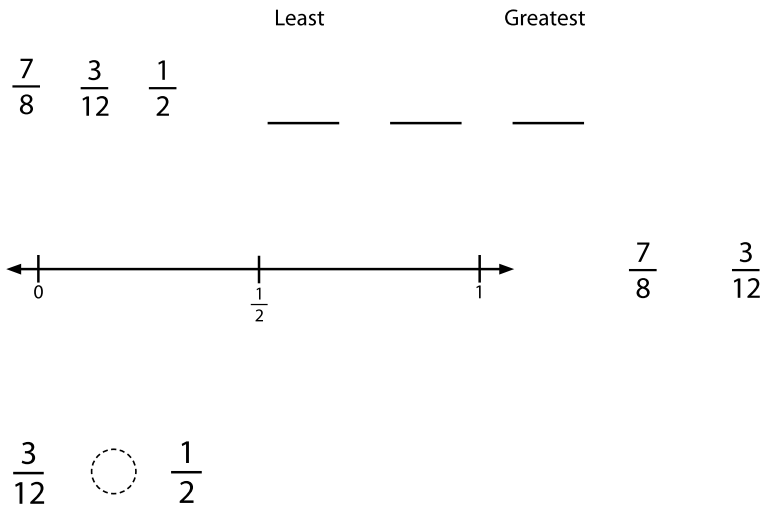
$1\frac{1}{5}$



$\frac{3}{4}$

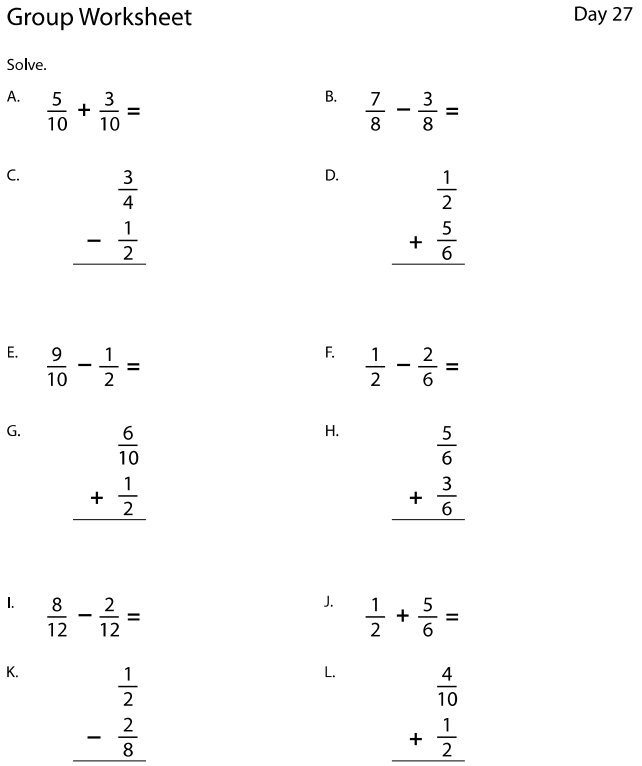
Building Magnitude Understanding

- Relating magnitude activities
- Use the same three fraction for each magnitude activity
  - Comparing
  - Ordering
  - Number line



Fraction Skill: Computation

- Limited instruction for computation
- Discussed denominators needed to be the same; demonstrated with fraction circles
- Explained an equivalent fraction needed to be written to add correctly



Group Worksheet

Day 27

Solve.

A. $\frac{5}{10} + \frac{3}{10} =$

B. $\frac{7}{8} - \frac{3}{8} =$

C. $\frac{3}{4} - \frac{1}{2}$

D. $\frac{1}{2} + \frac{5}{6}$

E. $\frac{9}{10} - \frac{1}{2} =$

F. $\frac{1}{2} - \frac{2}{6} =$

G. $\frac{6}{10} + \frac{1}{2}$

H. $\frac{5}{6} + \frac{3}{6}$

I. $\frac{8}{12} - \frac{2}{12} =$

J. $\frac{1}{2} + \frac{5}{6} =$

K. $\frac{1}{2} - \frac{2}{8}$

L. $\frac{4}{10} + \frac{1}{2}$

Fraction Skill: Improper to Mixed

- Demonstrate with fraction circles
- Relied on addition skills
- Limited whole number to 1
- Improper fractions < 2

Writing Mixed Numbers as Improper Fractions

Mixed Number: $1\frac{1}{2} = 1 \text{ and } \frac{1}{2}$

STEP 1:
Write a fraction equal to 1 with the same denominator $\frac{2}{2}$

STEP 2:
Answer to Step 1 plus proper fraction $\frac{2}{2} + \frac{1}{2} = \frac{3}{2}$

Your New Improper Fraction!

Writing Improper Fractions as Mixed Numbers

Improper Fraction: $\frac{3}{2}$

STEP 1:
Write a fraction equal to 1 with the same denominator $\frac{2}{2}$

STEP 2:
Subtract $\frac{3}{2} - \frac{2}{2} = \frac{1}{2}$

STEP 3:
Answer to STEP 2 plus 1 $\frac{1}{2} + 1 = 1\frac{1}{2}$

Your New Mixed Number!

Fluency: Magnitude Understanding

- Meet or beat your score!
- Looking at progress over three days—totals are graphed daily
- Activity resets after three days

Activity: Two Flashcard Types

- Single flashcards: 2 min. on clock
  - State whether fraction is equal to ½ or not.
  - State whether fraction is proper, improper or mixed.
- Compare flashcards: 2 min. on clock
  - State which fraction is bigger.
  - If correct, move on; if incorrect, state correct answer explaining rule.
  - Fraction comparison types increased in difficulty as lessons progressed.

Embedded Motivation System

- Students have three ways to earn fraction money
- On-task behavior
  - Unidentified intervals, group contingency
- Solving problems correctly
  - Last activity of the day
- Meeting or beating fluency score
  - Tutors were instruction to give bonus money to increase focus as needed based on group needs

Embedded Motivation System (cont.)

- Denominations of dollars include the following:
  - Whole dollars
  - Half dollars
  - Quarter dollars
- The Fraction Store opens every three days with prizes at various price points: $1, $7, $13, $20.
- Students can choose to save or spend each time store opens.

Results From Three Years of Research

Assessment	Tutoring vs Control (Y1)
Comparing	1.82
Number Line	1.14
NAEP	0.94
Calculations	2.51

Assessment	Fluency vs Control (Y2)	Conceptual vs Control (Y2)
Number Line	0.99	0.80
NAEP	0.60	0.63
Calculations	1.12	1.13

Assessment	M – WP vs Control (Y3)	A – WP vs Control (Y3)
Number Line	1.10	0.81
NAEP	0.44	0.33
Calculations	1.22	1.70

Implications for Intervention

- Students increased their ability to reason about fraction magnitude (number line results across three years).
- Students performed well on procedural computation (i.e., addition and subtraction) after extensive experience working on magnitude activities (calculation results across three years).
- We specifically addressed some of the common misconceptions or whole number bias that Russell spoke about:
  - Infinite equivalencies, numeral/value differences when in numerator versus denominator, a fraction is one number
  - See NAEP results across three years.

Intensive Intervention With Fractions

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at American Institutes for Research

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Response to Intervention (RTI) in Mathematics

- Tier 1
  - Evidence-based mathematics intervention

We need lots of work here.

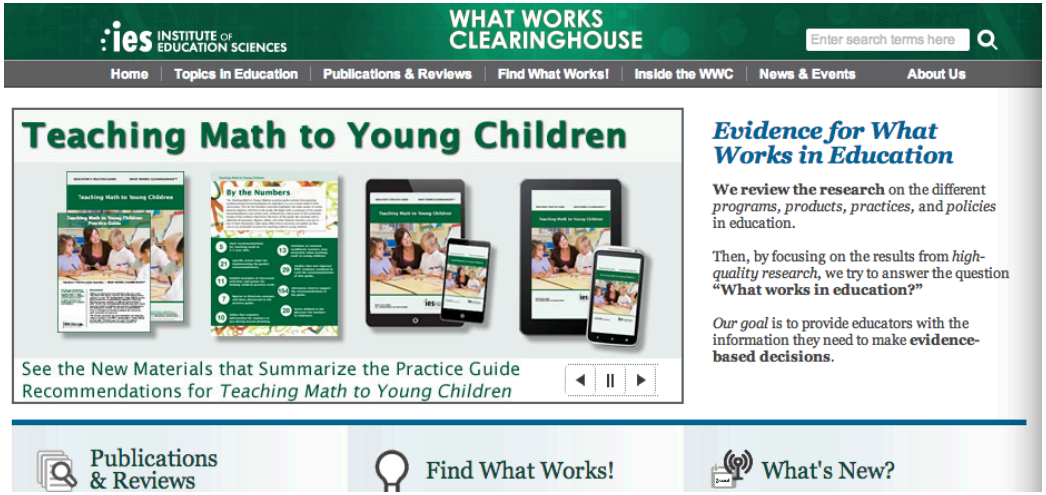
- Tier 2
  - Evidence-based mathematics intervention

We know quite a bit/some here.

- Tier 3
  - Evidence-based interventions mixed with intensive intervention

We do some good work here, but we need to help nonresponders.

Tier 1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We review the research on the different programs, products, practices, and policies in education.
Then, by focusing on the results from high-quality research, we try to answer the question "What works in education?"
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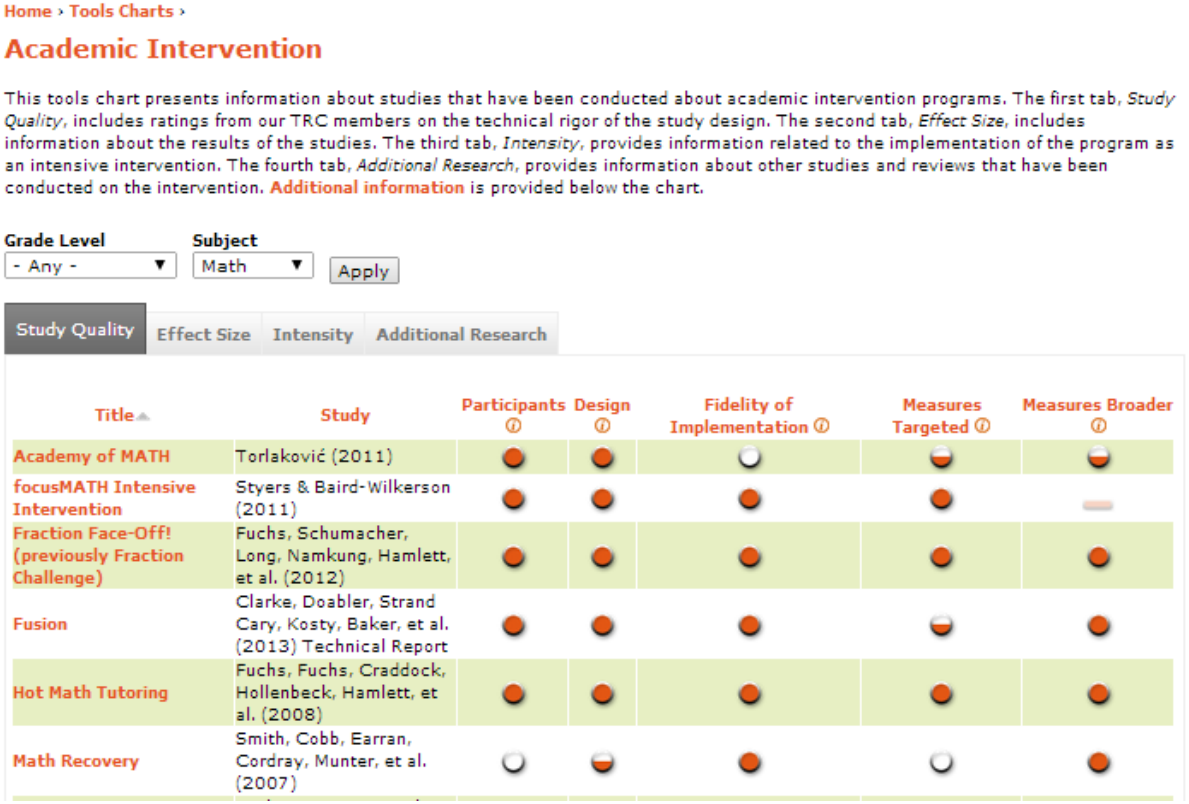
Program Reviews
Mathematics
Elementary
Middle/High School
Effectiveness of Technology
Which educational programs have been successfully evaluated in valid research?

Tier 2



Home › Tools Charts ›

Academic Intervention

This tools chart presents information about studies that have been conducted about academic intervention programs. The first tab, *Study Quality*, includes ratings from our TRC members on the technical rigor of the study design. The second tab, *Effect Size*, includes information about the results of the studies. The third tab, *Intensity*, provides information related to the implementation of the program as an intensive intervention. The fourth tab, *Additional Research*, provides information about other studies and reviews that have been conducted on the intervention. Additional information is provided below the chart.

Grade Level: - Any - | Subject: Math | Apply

Study Quality | Effect Size | Intensity | Additional Research

Title	Study	Participants	Design	Fidelity of Implementation	Measures Targeted	Measures Broader
Academy of MATH	Torlaković (2011)					
focusMATH Intensive Intervention	Styers & Baird-Wilkerson (2011)					
Fraction Face-Off! (previously Fraction Challenge)	Fuchs, Schumacher, Long, Namkung, Hamlett, et al. (2012)					
Fusion	Clarke, Doabler, Strand Cary, Kosty, Baker, et al. (2013) Technical Report					
Hot Math Tutoring	Fuchs, Fuchs, Craddock, Hollenbeck, Hamlett, et al. (2008)					
Math Recovery	Smith, Cobb, Earran, Cordray, Munter, et al. (2007)					

Other Tier 2 interventions may be listed on the WWC or BEE.

http://www.intensiveintervention.org/chart/instructional-intervention-tools

Tier 3

Intensive intervention addresses *severe and persistent* learning or behavior difficulties. Intensive intervention should be:

- Driven by data
- Characterized by increased intensity (e.g., smaller group, expanded time) and individualization of academic instruction and/or behavioral supports

What Is NCII's Approach to Intensive Intervention?

- Data-Based Individualization (DBI): A systematic method for using data to determine when and how to provide more intensive intervention
- Origins in data-based program modification/experimental teaching were first developed at the University of Minnesota (Deno & Mirkin, 1977) and expanded upon by others (Capizzi & Fuchs, 2005; Fuchs, Deno, & Mirkin, 1984; Fuchs, Fuchs, & Hamlett, 1989).
- It is a process, not a single intervention program or strategy.
- It is not a one-time fix but an ongoing process comprising intervention and assessment adjusted over time.

DBI Rests on Six Assumptions

1. Validated programs are not universally effective programs; 3 percent to 5 percent of students need more help (Fuchs et al., 2008; NCII, 2013).
2. Students with intensive needs often require 10–30 times more practice than peers to learn new information (Gersten et al., 2008).
3. Students with disabilities requiring special education need specially designed instruction to progress toward standards.
4. A data-driven, systematized approach can help educators develop programs likely to yield success for students with intensive needs.
5. DBI is a distinctively different and more intensive approach to intervention, compared to primary prevention's (Tier 1's) core program and secondary prevention's (Tier 2's) validated, supplementary programs (NCII, 2013).
6. In a longstanding program of field-based randomized control trials, DBI has demonstrated improved reading, mathematics, and spelling outcomes, compared to business-as-usual special education practice (e.g., Fuchs et al., 1989).

How Is NCII Addressing the National Need for Intensive Intervention?

- Operationalized DBI, a systematic, data-driven approach to intensive intervention
- Product development to articulate components of DBI
- Intensive technical assistance made up of ongoing on-site and distance training and implementation coaching
- Capacity building through local education agencies and regional or state technical assistance networks
- Connecting research to technical assistance with assessment and intervention tools charts
- Rigorous technical assistance evaluation
- Efforts to raise awareness, including targeted and universal technical assistance

Intensive Intervention With Fractions

National Center on
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at American Institutes for Research

Fraction Magnitude

Common Core State Standards Addressed: Extend understanding of fraction equivalence and ordering.

4.NF.2. Compare two fractions with different numerators and different denominators, for example, by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, for example, by using a visual fraction model.

Activity One: Comparing Fractions With Different Denominators

Purpose: To compare fraction magnitude between two fractions by finding common denominators.

Principles of Intensive Intervention:

- Provide concrete learning opportunities (including use of manipulatives).
- Provide explicit error correction and have the student repeat the correct process.
- Use precise, simple language to teach key concepts or procedures.
- Use explicit instruction and modeling with repetition to teach a concept or demonstrate steps in a process.

Materials (available for download from NCII):

Comparison flashcards (see Supplemental Materials section)

Multiplication chart (optional; see Supplemental Materials section)

Fraction tiles or fraction circles for justifying conclusions (see Supplemental Materials section)

Worksheet: Fraction Magnitude: Comparing Fractions With Different Denominators

Worksheet: Scaffolded Fraction Magnitude: Comparing Fractions With Different Denominators

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AIR

IDEAs that Work

Worksheet

Fraction Magnitude: Comparing Fractions With Different Denominators

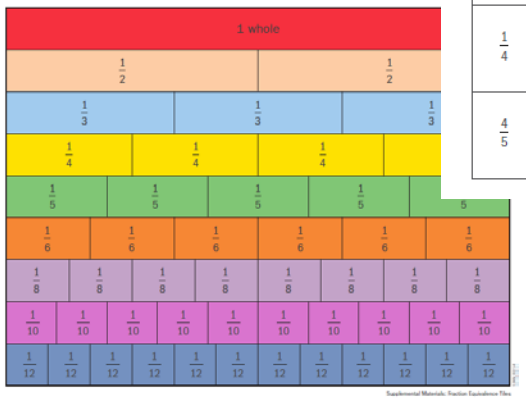
Objective: Given two fractions, compare them using greater than (>), less than (<), or equal to (=). Write the fractions with a common denominator when necessary.

Note: If the student struggles with this worksheet, try the Scaffolded Fraction Magnitude Worksheet.

Directions:

1. Look at the two fractions. Can you compare them or do you need to find a common denominator?
2. Multiply to find a common denominator, if necessary.
3. Compare the fractions using the greater than (>), less than (<), or equal to (=) symbol.
4. Check your work with the fraction tiles.
   a. If your answer is correct, draw a check mark and move to the next problem.
   b. If your answer is incorrect, go back and fix your work.

Fraction 1	Fraction 2	Show Work	<, >, or =	Check With Tiles ✓
$\frac{1}{2}$	$\frac{3}{6}$			
$\frac{1}{4}$	$\frac{3}{8}$			



Directions:

1. Look at the two fractions. Can you compare them or do you need to find a common denominator?
2. Multiply to find a common denominator, if necessary.
3. Compare the fractions using the greater than (>), less than (<), or equal to (=) symbol.
4. Check your work with the fraction tiles.
   a. If your answer is correct, draw a check mark and move to the next problem.
   b. If your answer is incorrect, go back and check your work.

Fraction 1	Fraction 2	Show Work and Write the Common Denominator	Compare: <, >, or =
$\frac{2}{5}$	$\frac{3}{4}$		
$\frac{2}{3}$	$\frac{1}{3}$		
$\frac{5}{7}$	$\frac{1}{2}$		
$\frac{1}{4}$	$\frac{3}{8}$		
$\frac{4}{5}$	$\frac{7}{10}$		

Explicit Instruction

- Requires instructors to clearly teach the steps involved in solving mathematical problems
- May take the form of teaching students how to use manipulatives, teaching specific algorithms for solving computational problems, or teaching strategies for solving more advanced mathematical concepts

Teaching Vocabulary and Symbols

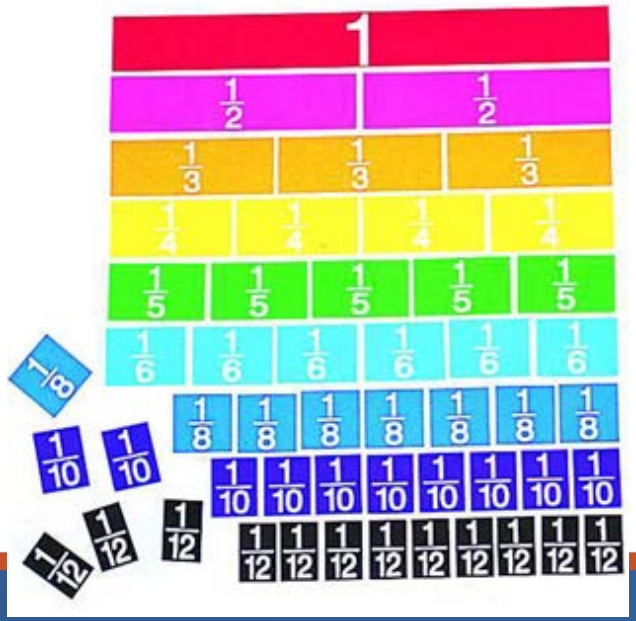
- Students with a strong mathematical vocabulary will have a better understanding of the skills being taught.
  - Use precise, simple language when teaching mathematical skills.
  - Integrate explicit teaching of vocabulary and mathematical symbols into all lessons.

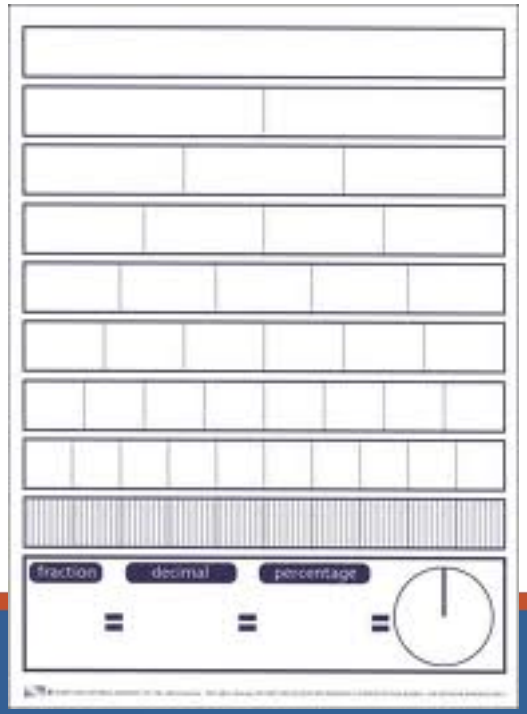
Graphic Organizers

- Helpful tools in mathematics that allow a great deal of information to be organized in one place
- Instructors explicitly teach how to use the graphic organizer and the content provided.
- Uses:
  - In place of extensive note-taking
  - To teach most mathematical concepts

Concrete-Representational-Abstract Model

- Model allows students to develop conceptual understanding before moving onto algorithms, rather than simply memorizing facts and algorithms.
- Three phases
  - Concrete
  - Representational
  - Abstract





Fluency Building

- Provide students with activities to build their fluency so that they are able to focus on higher level thinking skills as mathematical concepts become more complicated.
- Suggested activities
  - Timed worksheets
  - Flashcards
  - Learning centers
  - Computer software
  - Instructional games
  - Note: Many of these activities can be incorporated into peer tutoring activities.

Effective Questioning and Feedback

- Students who have difficulty in mathematics need many opportunities to respond to effective questions, explain their thinking, and receive feedback in order to improve their learning.

Error Analysis

- The process of analyzing student work to determine why they solved a problem incorrectly
  - Many errors can easily be detected, such as regrouping the ones rather than the tens, or adding denominators rather than finding common denominators.
  - Other errors that are specific to an individual's understanding of a process are more difficulty to identify.

What Is Happening in Other States?

Colorado

- RTI Implementation
  - http://www.cde.state.co.us/rti/toolsresourcesrti
- Multi-Tiered System of Supports (MTSS) Online Academy with training on mathematics intervention
  - http://www.cde.state.co.us/sites/default/files/CDE_MTSS_OnlineAcademy_Spring2014.pdf
- Extensive guidance on use of RTI/MTSS for specific learning disability identification
  - http://www.cde.state.co.us/cdesped/sd-sld
- Mathematics curriculum samples for Grades K–8 and algebra and geometry
  - http://www.cde.state.co.us/StandardsAndInstruction/Curriculum/Mathematics.asp

Michigan Integrated Behavior and Learning and Support Initiative (MiBLSi)

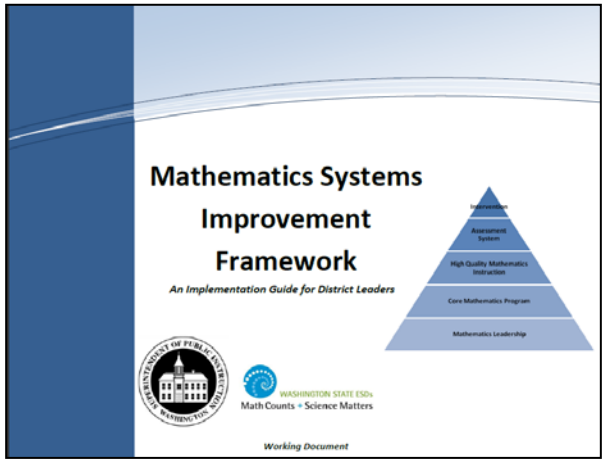
- MTSS in mathematics is an emerging work area pilot phase
- Long-range vision for how mathematics will be integrated into MiBLSi's model involves three main components.
- The data, systems, and practices for reading, mathematics and behavior will be fully integrated within the district cohort model.
- MiBLSi's model that integrates mathematics will include supports at Tiers 1, 2, and 3 with an emphasis on building strong Tier 1 foundations.
- The model will focus on early intervention and prevention for Grades K–5.
- http://miblsi.cenmi.org/EmergentWork/MathMTSS.aspx

Rhode Island

- Office of Special Education Programs State Personnel Development Grant with emphasis on integrating MTSS systems to incorporate academics and behavior, including mathematics
- Piloting implementation of middle school RTI in mathematics with initial evidence of positive impact
- RTI Technical Assistance Project:
  - http://www.ritap.org/rti/about/overview.php

Washington (state)

- Mathematics Systems Improvement Framework organized around an RTI structure:
  - Mathematics Leadership
  - Core/Tier I Mathematics Program
  - High-Quality Mathematics Instruction
  - Mathematics Assessment System
  - Tier II and Tier III Mathematics Intervention

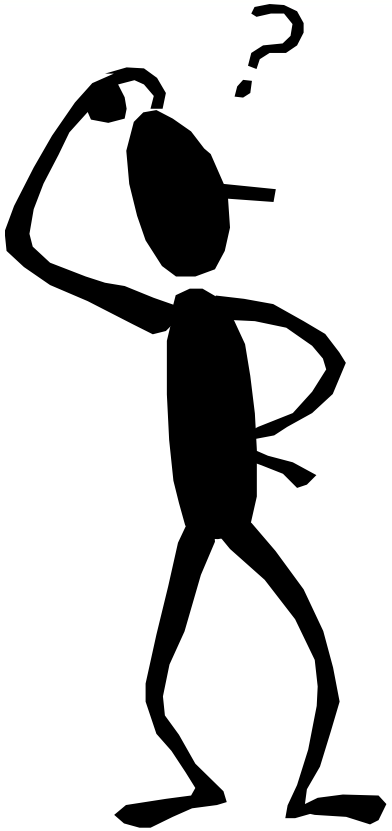


- http://www.k12.wa.us/Mathematics/SystemsImprovement.aspx

Wisconsin

- State RTI Center
- RTI process required for specific learning disabilities eligibility
- Mathematics initiatives
  - Universal mathematics screening
    - http://www.wisconsinrticenter.org/assets/files/Screener%20survey%20results/Universal%20Math%20Screener%20results.pdf
  - Intervention
    - http://www.wisconsinrticenter.org/assets/files/Mathematics_interventions.pdf
  - Reviewing universal mathematics instruction (with resources)
    - http://www.wisconsinrticenter.org/math.html

Questions



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  - NCSI@wested.org

Additional Information on Research on Fractions:

Shellenbarger, S. (2013). Why Are Fractions Key to Future Math Success? The Wall Street Journal. http://on.wsj.com/15rlupS

Sparks, S.(2013, July 18). Federal Research Suggests New Approach to Teaching Fractions. *Education Week.* *http://www.edweek.org/ew/articles/2013/07/18/37fractions.h32.html?tkn=TZQF7yozo%2FHWKGzn2KNa%2B63GIKKpFCTtESVi&cmp=clp-edweek*

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Siegler, R. S., Duncan, G. J., Davis-Kean, P. E., Duckworth, K., Claessens, A., Engel, M., Susperreguy, M. I., & Chen, M. (2012). Early predictors *of high school mathematics. http://www.psy.cmu.edu/~siegler/Siegler-etal-PsySci12.pdf*

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