National Center on
**INTENSIVE INTERVENTION**
at American Institutes for Research

# Teaching Basic Facts:
# Considerations for Instruction

## Purpose and Overview of Guide

The purpose of this guide is to provide strategies and materials for developing and implementing lessons for students who need intensive instruction in the area of **basic facts.** Special educators, mathematics interventionists, and others working with students struggling in the area of basic facts find this guide helpful.

Within college- and career-ready standards, Basic Facts are typically taught in grades K-4. This guide may be used as these concepts are introduced or with students in higher grade levels who continue to struggle with the concepts. Sample activities, worksheets, and supplemental materials also accompany this guide and are available for download at http://www.intensiveintervention.org.

The guide is divided into four sections:

1. Sequence of skills as defined by college- and career-ready standards.
2. A list of important vocabulary and symbols.
3. A brief explanation of the difficulties students may have with basic facts.
4. Suggested strategies for teaching basic facts.

## Sequence of Skills – College- and Career-Ready Standards

(Numbers in parentheses represents the grade level of the standard.)

### Addition and subtraction concepts:

- Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations. (K)
- Solve addition and subtraction word problems within 10. (K)
- Decompose numbers less than or equal to 10 into pairs in more than one way. (K)
- For any number from 1 to 9, find the number that makes 10 when added to the given number. (K)
- Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions. (1)

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- Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions. (2)

## Add and subtract:

- Fluently add and subtract within 5. (K)
- Apply properties of operations as strategies to add and subtract. (1)
- Understand subtraction as an unknown-addend problem. (1)
- Relate counting to addition and subtraction. (1)
- Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. (1)
- Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. (1)
- Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. (1)
- Fluently add and subtract within 20 using mental strategies. (2)
- By end of Grade 2, know from memory all sums of two one-digit numbers. (2)

## Multiplication and division concepts:

- Determine whether a group of objects (up to 20) has an odd or even number of members; write an equation to express an even number as a sum of two equal addends. (2)
- Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. (2)
- Interpret products of whole numbers. (3)
- Interpret whole-number quotients of whole numbers. (3)
- Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities. (3)
- Interpret a multiplication equation as a comparison. (4)
- Represent verbal statements of multiplicative comparisons as multiplication equations. (4)

## Multiply and divide:

- Determine the unknown whole number in a multiplication or division equation relating three whole numbers. (3)

- Apply properties of operations as strategies to multiply and divide. (3)
- Understand division as an unknown-factor problem. (3)
- Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division. (3)
- By the end of Grade 3, know from memory all products of two one-digit numbers. (3)
- Multiply or divide to solve word problems involving multiplicative comparison. (4)

## Vocabulary and Symbols

The following terms are important for students to understand when working with basic facts.

| | | |
|---|---|---|
| **Add:**<br>To combine amounts. | **Addend:**<br>An amount combined to another amount in an addition problem.<br><br>addend + addend = sum | **Sum:**<br>The total in an addition problem.<br><br>addend + addend = sum |
| **Plus sign:**<br>+ | **Subtract:**<br>To find the difference between two amounts; to take away. | **Minuend:**<br>The starting amount in a subtraction problem.<br><br>minuend – subtrahend = difference |
| **Subtrahend:**<br>The take away amount in a subtraction problem.<br><br>minuend – subtrahend = difference | **Difference:**<br>The end result in a subtraction problem.<br><br>minuend – subtrahend = difference | **Minus sign:**<br>– |
| **Multiply:**<br>To increase an amount a number of times. | **Factor:**<br>An amount multiplied by another amount.<br><br>factor × factor = product | **Product:**<br>The end result in a multiplication problem.<br><br>factor × factor = product |
| **Multiplication sign:**<br>X<br>Sometimes, * is used. | **Divide:**<br>To break an amount into equal groups. | **Dividend:**<br>The starting amount in a division problem.<br><br>dividend ÷ divisor = quotient |
| **Division:**<br>The number of groups in a division problem.<br><br>dividend ÷ divisor = quotient | **Quotient:**<br>The end result in a division problem.<br><br>dividend ÷ divisor = quotient | **Division sign:**<br>÷<br>Sometimes, / or a fraction bar is used. |
| **Additive**<br>Problem types relating to addition and subtraction. | **Multiplicative:**<br>Problem types relating to multiplication and division. | **Equal sign:**<br>=<br>Should be interpreted as "the same as." |

## Common Areas of Difficulty

**Prerequisite skills not mastered:**

- Knowledge of numbers and what number represent.
- Counting.

**Specific Basic Facts skills:**

- Understanding symbols.
- Fluency with facts.
- Strategies to calculate answers.

## About the Basic Facts

- *Addition:* 100 facts. Single-digit addend + single-digit addend = sum.
  - 3 + 4 = 7; 9 + 8 = 17
- *Subtraction:* 100 facts. Minuend – single-digit subtrahend = single-digit difference.
  - 8 – 2 = 6; 13 – 9 = 4
- *Multiplication:* 100 facts. Single-digit factor × single-digit factor = product.
  - 2 × 7 = 14; 8 × 6 = 48
- *Division:* 90 facts. Dividend ÷ single-digit divisor = single-digit quotient.
  - 9 ÷ 3 = 3; 56 ÷ 7 = 8

## Developing Conceptual Understanding

**Manipulatives** can be used to help students understand the concepts behind the basic facts. Examples of manipulatives include: Unifix cubes, plastic clips, chips, or dominoes.

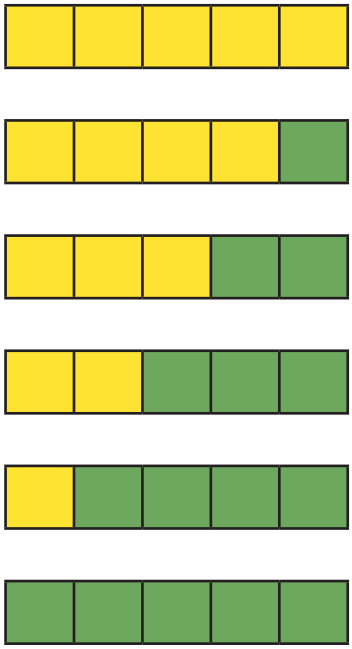
## Activities and Strategies Related to Specific Standards

**Represent addition and subtraction. (K)**

- Use objects (e.g., chips, fingers) to show two groups. Put the two groups together.
- Use objects (e.g., chips, fingers) to show one group. Take an amount away from the group.
- Tell stories to show addition (i.e., putting together, adding on).
- Tell stories to show subtraction (i.e., taking away, comparing).
- Use a number line. Move forward for addition. Move backwards for subtraction. (Also, compare the difference between two numbers for subtraction.)

**Decompose numbers less than or equal to 10 into pairs in more than one way. (K)**

- Use two different colors of the same manipulative to show all possible combinations of a specific number.

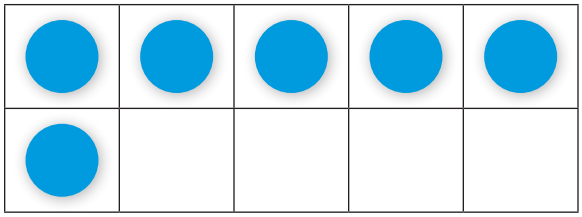


| | |
|---|---|
| 5 + 0 = 5 | 5 – 5 = 0 |
| 4 + 1 = 5 | 5 – 4 = 1 |
| 3 + 2 = 5 | 5 – 3 = 2 |
| 2 + 3 = 5 | 5 – 2 = 3 |
| 1 + 4 = 5 | 5 – 1 = 4 |
| 0 + 5 = 5 | 5 – 0 = 5 |

- Demonstrate commutative property of addition; the order of the addends does not matter. For example, 3 + 1 is the same as 1 + 3.

**For any number from 1 to 9, find the number that makes 10 when added to the given number. (K)**

- Use a Tens Frame and manipulatives to teach all possible combinations that make 10.



- Use a set of 10 clips or 10 cubes to teach all combinations that make 10.
- Use fingers. Start with some (e.g., 3) fingers held up. Count up to 10.
- Learn operation symbols (plus, minus, and equal signs) and how to use equation notation.

**Solve addition and subtraction word problems within 10. (K)**
**Use addition and subtraction within 20 to solve word problems. (1)**
**Use addition and subtraction within 100 to solve one- and two-step word problems involving situations. (1)**

- Present word problems written and orally. Provide situations that involve:
  - Adding to
    *"Maisie has 4 buttons. She buys 3 buttons at the store. How many buttons does Maisie have now?"*
  - Taking from
    *"Maisie had 9 buttons, and then her brother took 2 of them. How many buttons does Maisie have left?"*
  - Putting together
    *"Maisie has 5 buttons. Jamey has 4 buttons. How many buttons do they have altogether?"*
  - Comparing
    *"Maisie has 8 buttons and Jamey has 2 buttons. How many fewer buttons does Jamey have than Maisie?"*
- Use stories where the unknown is in all positions.
  - For the comparing problem from above:
    - *"Maisie has 8 buttons and Jamey has 2 buttons. How many fewer buttons does Jamey have than Maisie?" (unknown: difference)*
    - *"Maisie has 8 buttons. She has 6 more buttons than Jamey. How many buttons does Jamey have?" (unknown: subtrahend)*

- *"Jamey has 2 buttons. Maisie has 6 more buttons than Jamey. How many buttons does Maisie have?" (unknown: minuend)*

- Act problems out with objects or students.

**Determine whether a group of objects (up to 20) has an odd or even number of members. (2)**

- Practice skip counting by twos.
  - Start at 0 to skip count even numbers.
  - Start at 1 to skip count odd numbers.

**Write an equation to express an even number as a sum of two equal addends. (2)**

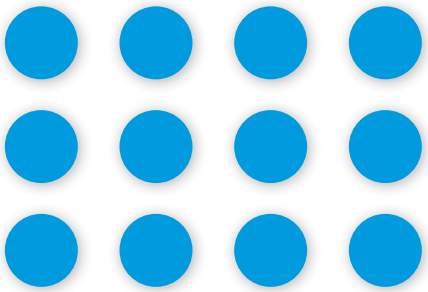
- Teach the addition doubles with rhymes or chants.
- Look at doubles patterns on an addition chart.

| + | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 3 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| 4 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 5 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| 6 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| 7 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| 8 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| 9 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
| 10 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |

**Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns. (2)**

Write an equation to express the total as a sum of equal addends. (2)

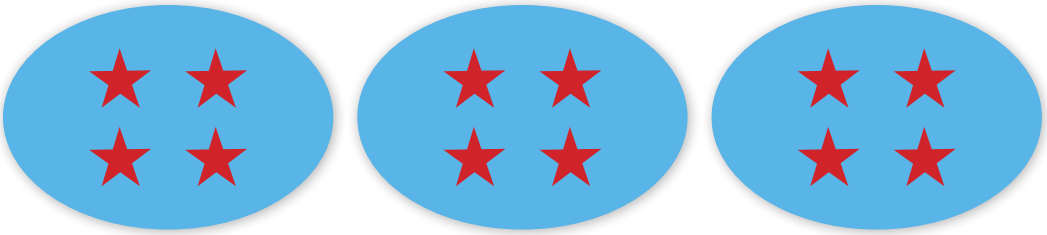
- Arrange objects in arrays and skip count rows or columns. This helps with learning multiplication as repeated addition.



- Learn operation symbols (multiplication, division, and equal signs) and how to use equation notation.

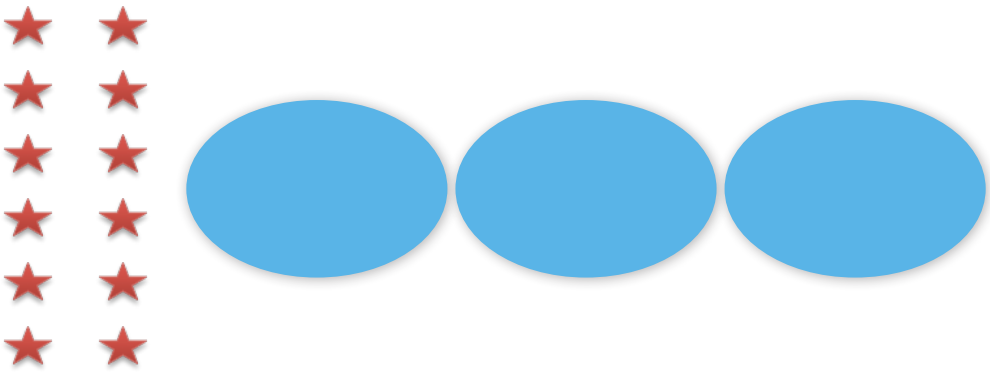
**Interpret products of whole numbers. (3)**

- Use objects (e.g., chips and plates) to show a number of groups and the number within each group.



**Interpret whole-number quotients of whole numbers. (3)**

- Use objects (e.g., chips and plates) to show an amount divided evenly into groups.



**Use multiplication and division within 100 to solve word problems. (3) Multiply or divide to solve word problems. (4)**

- Present word problems written and orally. Provide situations that involve the following:
  - Equal groups
    "*Lincoln has 5 buckets with 4 toy cars in each bucket. How many toy cars does Lincoln have?"*
  - Comparison
    *"Lincoln has 4 toy cars. Roscoe has 7 times more cars than Lincoln. How many toy cars does Roscoe have?"*
  - Combinations
    *"Lincoln has 4 shirts and 2 pairs of pants. What are all the combinations of shirts and pants?"*
- Use stories where the unknown is in all positions of the problem.
  - For the equal groups problem from above:
    *"Lincoln has 5 buckets with 4 toy cars in each bucket. How many toy cars does Lincoln have?"* (unknown: product; 5 × 4 = ?)

    *"Lincoln has 20 toy cars. He places the same number of cars in 5 buckets. How many cars are in each bucket?"* (unknown: factor—number in each group; 5 × ? = 20)

    *"Lincoln has 20 toy cars. He wants to put 4 toy cars into each bucket. How many buckets does Lincoln need?"* (unknown: factor—number of groups; ? × 4 = 20)
- Act problems out with objects or students.

**Fluently add and subtract within 5. (K)**
**Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. (1)**
**Fluently add and subtract within 20 using mental strategies. (2)**
**By end of Grade 2, know from memory all sums of two one-digit numbers. (2)**

- Teach strategies for solving addition problems. Say, "Put the larger number in your head. Count on the smaller number by holding up a finger for each count. The sum is the last number you say."
- Teach strategies for solving subtraction problems. Say, "Put the subtrahend in your heard. Count up to the minuend. The difference is the number of counts."
- If a student answers incorrectly, encourage them to count to calculate the answer.
- Use number lines or addition tables if necessary.
- Practice with flashcards. Provide immediate feedback to incorrect facts.
- Practice within fact families. (3 + 4, 4 + 3, 7 – 3, 7 – 4)
- Practice with timed and untimed paper activities. Provide immediate feedback to incorrect facts.

**Apply properties of operations as strategies to add and subtract. (1)**

- Teach the commutative property of addition. Teach how the commutative property can help students solve addition and subtraction facts. Say, "If you know 4 + 5 = 9, you also know that 5 + 4 = 9."

**Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. (1)**

- Show students a balance scale. Place objects on each side of the balance to represent addition and subtraction equations. Emphasize that both side of the balance must be the same.

**Understand subtraction as an unknown-addend problem. (1)**
**Relate counting to addition and subtraction. (1)**

- Teach the reciprocal property of addition and subtraction. Say, "If you know 4 + 5 = 9, you know 9 – 4 = 5 and 9 – 5 = 4."
- Teach of thinking of subtraction as, "What number do I add to get to __?" "Let's see. 14 – 9 = __. What number can I add to 9 to get 14? I start with 9 and count up. 10, 11, 12, 13, 14. I added 5. 14 minus 9 is 5."

**Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. (1)**

- Introduce unknowns within stories and using manipulatives. For example, with a cup and chips: "I have 5 chips. There are some chips in the cup. If I have 8 chips altogether, how many chips are in the cup?"

**Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division. (3)**
**By the end of Grade 3, know from memory all products of two one-digit numbers. (3)**

- Teach strategies for solving multiplication problems as repeated addition. Say, "7 times 3. That's 7 counted 3 times. 1, 2, 3, 4, 5, 6, 7. 8, 9, 10, 11, 12, 13, 14. 15, 16, 17, 18, 19, 20, 21. 7 times 3 is 21."
- If a student answers incorrectly, encourage them to count to calculate the answer.
- Use number lines or multiplication tables.
- Practice with flashcards. Provide immediate feedback to incorrect facts.
- Practice within fact families. (6 × 4, 4 × 6, 24 ÷ 6, 24 ÷ 4)
- Practice with timed and untimed paper activities. Provide immediate feedback to incorrect facts.

**Determine the unknown whole number in a multiplication or division equation relating three whole numbers. (3)**

- Introduce unknowns within stories and using manipulatives.

**Apply properties of operations as strategies to multiply and divide. (3)**

- Teach the commutative property of multiplication. Teach how the commutative property can help students solve multiplication and division facts. Say, "If you know $3 \times 8 = 24$, you also know that $8 \times 3 = 24$."

**Understand division as an unknown-factor problem. (3)**

- Teach the reciprocal property of multiplication and division. Say, "If you know $7 \times 5 = 35$, you know that $35 \div 7 = 5$ and $35 \div 5 = 7$."
- Teach of thinking of division as, "What number do I multiply to get to __?" "Let's see. 21 divided by 7. What number can I multiply to 7 to get to 21? I can count by 7. 7, 14, 21. I counted 7 3 times. 21 divided by 7 is 3."

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